9 Default invariance
A naïve category theory of law and finance

Joseph Tanega

Introduction
Default invariance is the idea that default does not change at any scale of law and finance. Default is a conserved quantity in a universe where fundamental principles of law and finance operate. It exists at the micro-level as part of the fundamental structure of every financial transaction, and at the macro-level, as a fixed critical point within the relatively stable phases of the law and finance cycle. A key point is that default is equivalent to maximising uncertainty at the micro-level and, at the macro-level, is equivalent to the phase transition where unbearable fluctuations occur in all forms of risk transformation, including maturity, liquidity and credit. As such, default invariance is the glue that links the micro and macro structures of law and finance. In this essay, we apply naïve category theory (NCT), a type of mapping logic, to these types of phenomena. The purpose of using NCT is to introduce a rigorous (but simple) mathematical methodology to law and finance discourse and to show that these types of structural considerations are of prime practical importance and significance to law and finance practitioners. These mappings imply a number of novel areas of investigation. From the micro-structure, three macro-approximations are implied. These approximations form the core analytical framework which we will use to examine the phenomena and hypothesise rules governing law and finance. Our observations from these approximations are grouped into five findings. While the entirety of the five findings can be encapsulated by the three approximations, since the intended audience of this paper is the non-specialist in law, finance and category theory, for ease of access we will illustrate the use of the mappings with relatively common concepts drawn from law and finance, focusing especially on financial contracts, derivatives, Shadow Banking, credit rating agencies and credit crises.

In this brief essay, we present an application of naïve category theory (NCT) on law and finance discourse. Given the constraint of space and the very large number of correspondences and points of departure between these vast fields, we can only provide an indicative summary in
the form of approximations and findings. We present NCT applications in terms of three approximations of the law and finance universe, and we condense our observations of the logical implications of these approximations into five findings, two of which focus on the micro-structure, one on the bridge between the micro- and macro-structure, and two on the macro-structure. Since we presume our intended audience includes those who know something of law and finance but little or nothing of category theory, our attitude in this essay is not to present theory for theory’s sake, but rather pretend a serious commitment of theory to practice. One more warning before we begin in earnest. Essentially, NCT is a visual language with its own caricatures. For this reason, we ask for your indulgence in what may seem to be a disproportionate number of diagrams to illustrate rather trivial-to-profound truths. This is part of the category theoretic culture, where the “meme” if you will is in the diagram (sketch or esquisse), and out of merciful convenience, we learn that extraordinarily deep and complex relationships which in English would be unintelligible gibberish can be communicated by pointing to a diagram with arrows. While the points made in this essay regarding particular phenomena of law and finance, such as financial derivatives, Shadow Banking, credit rating agencies and credit crises are entirely serious, we shall take a breezy attitude to NCT concepts and define them as we need them, providing technical references in the endnotes. How shall we begin? For those with a bit of category theory background, I would recommend skipping Section 2 and diving into Section 3 where we correct the work of three Nobel Laureates of Economics – Arrow, Debreu and Sharpe. For those looking for a street-level entrance to the fundamentals of NCT, we recommend taking the London Underground at Victoria Station by way of analogy in the next section.

NCT explained: the Circle Line as the commuting square

Suppose on the Victoria Station platform in the Tube of London, a stranger pointing at the track asks you, “which way to Liverpool Street?” Suppose further, you answer, “Either way.” This is the correct answer if you both happen to be on the Circle Line platform, but if the stranger has no idea of the Circle Line route, this answer could cause confusion and even more uncertainty. If the stranger, for example, presumes the Circle Line runs linearly then the answer, “either way” will make no sense at all to him. But if the stranger twigs that the Circle Line is some form of “circle”, then presto, the answer makes perfect sense. See Figure 9.1.

Note the intention of the stranger’s question is not to find out which way is farther, cleaner, or more convenient. The only relevant piece of information, which the stranger seeks, is the destination. It is important to keep in mind that the context is that of information and communication exchange and that the raison d’être of this general sphere of exchange is the
minimisation of uncertainty to certainty. Within this sphere of exchange, NCT and category theory in general become a supremely “result-oriented” methodology. The ancients called this teleology. Aristotle could be said to have invented teleology at least as a systematic logical concept but he did not have the simple technical tools that would have allowed him to realise a full-blown teleological system of explanations of reality. The arrows in the diagram are called arrows or morphisms, and for each morphism, there is an object at the blunt end of the arrow called the domain or source of the arrow, and the object at the sharp end of the arrow, called the target or codomain. From a philosophical perspective, you might consider each morphism with domain and codomain as a “mini-teleological system” and thus, mini-teleological systems linked up are just a macro-teleological system. When two arrows connect up such that they share the same object where the target of one arrow is the source of another arrow, then we have a composite. For example, in Figure 9.1, the morphism $g$ and morphism $f$ form a composite $gf$, that is, “$g$ following $f$”.

We can translate the diagram in Figure 9.1 into more economical notation. For example, we can say “the morphism $f$ from Victoria to Paddington [$f$: Victoria $\to$ Paddington] and the morphism $g$ from Paddington to Liverpool [$g$: Paddington $\to$ Liverpool] form a composite, $g$ following $f$, [$gf$] and the morphism $h$ from Bank to Liverpool [$h$: Bank $\to$ Liverpool] and the morphism $i$ from Victoria to Bank [$i$: Victoria $\to$ Bank] form a composite, $h$ following $i$ [$hi$].” Or, we can simply point at the diagram in Figure 9.1 and say, “the diagram commutes” and this finger-pointing-and-oral-declaration would be a perfectly legitimate and meaningful action amongst category theorists. The important part of the structure is the equivalence between the two composites, that is, $gf = hi$. Because of this equivalence, we say the “diagram commutes” and designate the diagram as a “commuting square”. To get a feel of what this mapping means in non-mathematical terms, we can take apart the mapping into some of its various parts and ask for some subjective associations.

Figure 9.1 How to get to Liverpool Street Station?
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Being lost

For example, looking at Figure 9.2, where the two morphisms, \( f \) and \( i \), share the same domain but have different codomains, we might ask, “How does the diagram make us feel?” Students asked this question in class usually say that the diagram gives them a sense of feeling “lost”, or “directionless”, or “separated.”

Intuitively, this wedge or span is technically called a projection and denotes a type of multiplication structure that is very well defined. However, from a subjective point of view, the diagram connotes an openness and separation. These subjective characterisations are interpretations of the syntactic structure. The syntactic structure does not change and in this sense it is a fundamental unit. The diagram could also be interpreted as a maximisation of uncertainty, since there appears to be no linkage between \( B \) and \( D \).

Being found

In sharp contrast to Figure 9.2, consider Figure 9.3 in which the two morphisms, \( g \) and \( h \), do not share the same domain but share the same codomain. Again, we might ask, “How does this diagram make us feel?” The answers to this question range across “secure”, “safe”, “certain” and “knowing where I am”.

![Figure 9.2 Feeling lost in a projection?](image1)

![Figure 9.3 Feeling safe, secure and certain in an injection?](image2)
Technically, the diagram in Figure 9.3 denotes an injection and a common vertex or shared end-object, the shared codomain. And rather technically, we call this sort of plural-objects-to-one-object morphisms, surjective, because each and every one of the objects of the codomain have at least one morphism from the domain. These morphisms also suggest a confirmation of the singular object. For our purposes, this type of structure indicates a single end-point object, and is very close to what we call a “terminal object”.

Later we shall see that the features of the shared codomain differentiate the types of models we can have about the operation of financial contracts. But this is getting ahead of ourselves. The destination itself, that is, the shared codomain in the above diagram can be thought of as a terminus in itself or as a kind of switching station where other morphisms connect. We shall discuss these features in depth in terms of terminal objects and coproducts under Finding 3 below.

The ideal as short-cut: the commuting triangle

Having understood that the Circle Line can be translated into a commuting square, we can also imagine a short-cut from Victoria Station to Liverpool Street that cuts the commuting square into two commuting triangles. See Figure 9.4 below.

Note the \( m \)-morphism is imaginary and ideal, because there is no physical Tube route directly from Victoria Station to Liverpool Street that corresponds to the \( m \)-morphism and yet we can describe it precisely. There is an isomorphism between the concept of a short-cut and something which may be built in reality, but more specifically, the \( m \)-morphism exists as a structure within our minds. It is interesting to note how the \( m \)-morphism is the hypotenuse of the Pythagorean Theorem which is “re-proved” by a slave boy under Socrates’ simple interrogation in the Platonic dialogue, the Meno. In a strong sense, the commuting square has a correspondence to a de facto reality while the \( m \)-morphism indicates some kind of de jure short-cut. This is an important distinction to keep in mind when we investigate financial derivatives contracts since the \( m \)-morphism incorporates an important universal concept in law and finance, variously called

![Figure 9.4 m-Morphism as an imaginary short cut](image-url)
“arbitrage”, “the law of one price”, or “fungibility”. Conceptually, it is better to think of “arbitrage” not as an object that exists per se as an ideal form, but rather as a morphism (an ideal process) that results in the equivalence that is assumed to underlie the simultaneous buying and selling of an asset.

And it is one of the major simplifications and insights of NCT to make the claim that the arbitrage function of all financial contracts can be thought of as an m-morphism. If this is hard to believe, just think why it is that the total volume and nominal size of derivatives contracts are so much bigger than the size of physical trade in the world, and that these contracts supposedly all net out. The netting-out function is simply an m-morphism. Derivatives contracts merely make reference to indexes and are in a strong sense synthetics that refer to end-points in reality, but are not themselves changed by the physical reality, since they exist in a de jure sense.

The m-morphism also allows us to consider the structure of trades that originate in the financial markets but are aimed to resolve challenges in the judicial and political sphere. For example, the legal structure of a collateralised debt obligation (CDO) is such that calls on collateral in case of default mean that the CDO in effect becomes worthless in case of bankruptcy since all collateral will be paid out prior to or at a bankruptcy event. It is also possible to structure financial derivatives so that particular political events, for example, change of regulation or government triggers payment of a certain fixed amount of cash. Another example is “COCOs”, that is, contingent capital bonds where an event of default converts the debt into equity. This apparently suicidal bond, which states clearly that a holder stands to lose their entire investment in case of default is extremely popular, since it pays a comparatively high yield and qualifies as tier 1 capital under Basel III. Basel III, in a word, is the international voluntary standard on capital adequacy of international active banks and includes stress testing and liquidity risk. From a regulatory point of view, tier 1 capital measures the financial strength of a bank since it includes both equity capital and disclosed reserves.

Thus, the m-morphism can be thought of as the derivative contract, which achieves the result of the shared codomain, whether it be in the market sphere, or judicial and political sphere. But we are getting ahead of ourselves again. Coming back to our visitor in London looking for the way from Victoria Station to Liverpool Street Station, once he comprehends the structure of the Circle Line, he may ask if there is a more direct journey. Even it does not exist, he may dream of it being built one day.

Some general philosophical considerations

The point of illustrating the commuting square with the Circle Line is to show how category theory diagrams are maps, which can have many interpretations, but they are all in a strong sense determinable by
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universals (i.e., by extremes). This raises questions about what we mean about interpretations in relation to these universal structures. A more “universalistic” interpretation of the commuting square is to conceive of money or financial contracts as a commuting square. See Figure 9.5 below.

These interpretations can be very loose to the point where they fail to be connected to the syntactic structure. However, the interpretations work well with the syntactic structure when different individuals can set aside their differences and agree about what the syntactic structure tells us about what to do, where to go, how to arrive and exactly what steps need to be taken to get from one agreed “location” to another. In other words, very much like a Tube map, the syntactic structures make sense in a network of communications. Keeping the syntactic structure of the Circle Line in our minds helps us negotiate and deal with the challenging realities of getting to our desired destination. How does this happen?

We can interrogate the map for its structure. And since the structure of the world of the Circle Line is embedded in the morphisms and objects of the commuting square, we have a one-for-one correspondence between four bare arrows and the four Tube routes, and between the four objects in the map and the four stations on the Circle Line. This is an isomorphism. More specifically, this invariant isomorphism also works for defining the essential structure of all financial contracts. It is amazing that so little notation – literally, eight squiggles on a page – can inform and communicate so much information about how to navigate in reality, posting extreme idealisations such as eliminating uncertainty and achieving certainty in the real world.

Figure 9.5 Thinking of financial contracts as extreme events
In a general sense, the essential knowledge we need of the world comes from our maps of the world.

The syntactic structure of NCT allows us to ask questions of universal structures and leaves it to us to find the meanings of these structures in the law and finance discourse. Thus, to find meanings of the syntactic diagrams in the world tests our understanding of the specific applicability of the syntactics. The five findings below are literally the result of implementing the methodology wherein the phenomenon in question, having been isolated, implies a particular structure by virtue of its translation into the language of NCT, i.e., of objects, morphisms, composites, diagram chasing, and so on.

If we can translate a phenomenon into category theory terms, then there may be some form of equivalence between the theoretical structure and the phenomenological structure in question. If there is a one-to-one correspondence going both ways from the components of the abstract mapping to the elements of the phenomenon, then we may have an isomorphism up to the level of uniqueness. In our example, of a commuting square to the mapping of the Circle Line, i.e., there is an exact correspondence between objects and morphisms of the two mappings. In a general determination by universals, isomorphism precedes any quantification and enables us to determine the identity and uniqueness of entities in terms of their relations to each other. We now leave this simple example of the Circle Line to see how NCT may help us understand more complex structures of law and finance.

The contingent financial contract as a fundamental unit of law and finance

So, what is the structure of law and finance? Specifically, our answer to this question begins with a reference to Sharpe’s article, “Nuclear Financial Economics” (Sharpe, 1993). Essentially, Sharpe took Arrow’s and Debreu’s concept of contingent claims (ibid., p. 2) to formulate the axioms of financial economics. We have taken the liberty to translate Sharpe’s version of Arrow and Debreu’s contingent claims model into category theory terms, dubbing it the Arrow-Debreu-Sharpe Model, or ADSM. ADSM may be thought of as the precise translation of contingent claims into financial contracts in terms of NCT. And it is this translation of NCT into ADSM that allows us to generate mappings of the law and finance universe at the micro-level. Sharpe considered Arrow’s and Debreu’s insights re contingent contracts of the time-claim world as the intellectual foundation to much of financial engineering, including:

- binomial models of asset returns,
- continuous time models of Black and Scholes,
• corporate and investment analysis,
• corporate governance, and
• importantly, financial calculations based on arbitrage-free versus investor preference and predictions. (Ibid., pp. 3–5)

By taking seriously what Sharpe referred to with “a bit of hyperbole” as “nuclear financial economics” (ibid.) and using the “abstract nonsense” gadgets of NCT such as “generalized element”, “commuting squares”, “commuting triangles”, “chain complexes”, “products and co-products”\textsuperscript{18} we can move readily from a micro to macro perspective and \textit{vice versa}. That is, by translating the ADSM into a NCT syntactic structure, we can get a lot more structure than our initial investment! And we can explain much more than what Sharpe had envisaged.

It may seem that we are merely extending the work of the three Nobel Laureates in Economics, but we are actually, correcting their work by placing it within a much larger theoretical framework. In a word, the ADSM presumes the certainty of payment in a real world but failed to take account of the invariance of default. For example, in the real world of law and finance, although we expect simple payment of financial contracts, we have what appear to be extraordinarily complex legal procedures (i.e., the judicial system) and political processes (i.e., the political system) that allow for the delay, injunction and immunity of non-payment, e.g., the re-structuring of financial transaction, the forgiveness of debt, the indeterminable re-hypothecation of collateralised pools of self-liquidating assets in the shadow banking system, or the sudden confiscation wrought by administrative orderly liquidation – to name but a few. Those who used ADSM also incorrectly quantified the event of non-payment by designating the value of non-payment as 0 – which at first seems perfectly reasonable. However, this is an error, since it fails to understand the initial commitment of the structure in question to the universal of “infinite contingency”, which is uncertainty. That is, the correct quantification scheme for an event of payment and non-payment for a financial contract that is a commuting square should be 0 and 1, respectively. Thus, uncertainty multiplied by an event of payment (i.e., 0) results in the elimination of uncertainty, and uncertainty multiplied by an event of non-payment (i.e., 1) results in a continuation of uncertainty. It is in this fundamental sense, that default (not-payment as per agreed time) continues uncertainty. Since default is potentially existent at the time of formation of the contract, and may eventuate in an “actual actuality”,\textsuperscript{19} it is invariant throughout the spheres of exchange of the Market System. This default invariance invades the Judicial and Political Systems, and speculatively, the Communications System, which translates the randomness of non-information into packets of completely certain information.\textsuperscript{20}
The serious commitment of theory to practice

An NCT of law and finance, if it is to be considered a theory at all, must be useful to practitioners in their perception and actions of daily practice, whether big or small. Hohfeld, the great legal theorist, set out this criterion when he established a jural relations theory, and that view is very much in keeping with our approach of building a bridge from practice to theory and a co-bridge from theory to practice. NCT no doubt will be useful to practitioners because the sweep of its mapping includes a much greater range of events facing practitioners. The implications and significance of NCT in relation to law and finance may be thought of in terms of the definition of category theory syntactic structures and their correspondent translations in law and finance discourse. Once we have isolated the phenomenon of law and finance for concentrated study, the NCT syntactic structures naturally arise from a reconstruction of the phenomenon (see Geroch, 1984, p. 1). We list these syntactic structures and their law and finance interpretations as the five findings, not because they are literally or merely five structures (there are many more!), but rather because certain regions of practice lend themselves to be organised in terms of particular types of syntactic structures.

Mapping the wild regions of law and finance: five findings

At a fairly high level of abstraction, we can think of the five findings as five sorters, where NCT is the morphism from the domain of raw data of law and finance to the codomain of areas of law and finance phenomena. All five findings are parts of the generalised element of law and finance discourse. See Figure 9.6.

Figure 9.6 makes explicit the slogan, “Show the phenomenon fits within naïve category theory, you get the rest of category theory for free.” The Five Findings act as sorters to the types of category theory structures that can be used by a law and finance practitioner. If the law and finance phenomenon is found as one of the five types of findings, then it is of course within the scope of a category proper under category theory.

As a mnemonic, we can think of the five findings as arrows forming lots of different syntactic structures, e.g.:

- four arrows forming a square,
- three arrows forming a triangle,
- linked arrows forming a longer arrow,
- an arrow circling back onto itself,
- a projection of two arrows,
- an injection of two arrows,
These austere structures have nothing to do with law and finance, except for the fact that for their abstract design, when specifically defined in terms of NCT, gives us a set of gadgets that we can in turn use to define certain patterns of phenomena in law and finance. It is important to note that we are not literally inputting new raw material in law and finance discourse, rather we are re-structuring the way we conceptualise phenomena in law and finance. Using Spivak’s analogy, when we use category theory, we are more concerned with the structure of the database than the raw data (Spivak, 2013, pp. 65, 98–9).

When we are aware that such structures exist in the abstract sense, they then naturally arise from focusing on transactions occurring between individuals engaged in law and finance. And it is the combined linkage of “abstract structure to observable artefact in law and finance” that we call a finding. The irony (which indeed is a deep-seated teleological bias) is that “we find what we are looking for so long as we know what we are looking for.” In summary form, the five findings can be stated in terms of the anchoring gadget together with a set of statements implied by the anchoring gadget as appears in Table 9.1, Box 9.1 and Table 9.2.
Table 9.1 Summary of the Five Findings

<table>
<thead>
<tr>
<th>Finding</th>
<th>NCT structures</th>
<th>Relations between NCT and law and finance</th>
<th>Law and finance meanings</th>
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<tbody>
<tr>
<td>1st</td>
<td>Commuting square</td>
<td>Commuting square ≡ Contingent financial contract</td>
<td>The commuting square is one of the fundamental structures of the law and finance universe. Each and every financial contract (specifically defined as having only one condition left to perform) has a commuting square structure. Contingent financial contract is the fundamental unit of the law and finance universe.</td>
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<tr>
<td>2nd</td>
<td>Commuting triangles</td>
<td>Commuting triangles ≡ Financial Derivatives Contracts $m$-morphism ≡ arbitrage = law of one price Lower right commuting triangle ≡ Epistemological route Upper left commuting triangle ≡ Ontological route</td>
<td>The commuting triangle is one of the fundamental structures of the law and finance universe. Low-risk derivatives and high-risk derivatives share the same diagonal $m$-morphism. No-risk derivatives are lower right commuting triangles of the commuting square. High-risk derivatives are higher left commuting triangles of the commuting square.</td>
</tr>
<tr>
<td>3rd</td>
<td>Shared Codomain as Terminal objects and coproducts</td>
<td>[Shared Codomain → Terminal Object → Pay] → Single Commuting Square [Shared Codomain → Not-Pay] → Multiple Commuting Square [Chain Complex of Risk Homology]</td>
<td>Three different types of shared codomain define three different types of structures that approximate the law and finance universe. 1st Approximation: Pay as the terminal object of the shared codomain defines the Arrow-Debreu-Sharpe Model, i.e., a single commuting square. 2nd Approximation: Not-Pay as the shared codomain defines an &quot;initial object&quot; of a risk homological structure which is a chain complex comprising adjacent commuting squares labelled in order from left to right, &quot;Markets system&quot;, &quot;Judicial system&quot;, &quot;Political system&quot; and &quot;Communications system&quot;.</td>
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</table>
3rd Approximation: Pay and Not-Pay as the shared codomain defines a coproduct; since we define the structure in category theory terms, we get the rest of category theory for free, which means that coproduct comes from a product, and that the product-coproduct structure comes from the distributivity law that in general implies a ring structure. The ring structure for our purposes is a cyclic matrix where default is invariant, such that if we exponentiate (P & ~P) by default, we have the following standard distributivity: D*[I][I] + D*[I][B] + D*[B][I] + D*[B][B] = D*[I][I] + [I][B] + [B][B] + [B][I]; in a word, default is invariant in different states of the world. The bracket notation “[ ]” and “{ }” is to indicate sets, and the “*” means “binary operation”. For more details concerning this notation, see Finding 5.

Single commuting square of financial contract means uncertainty of payment is annihilated by payment. The chain complex of adjacent commuting squares begins with Market system on the left, then Judicial system, then Political system and then Communications system on the right. The morphisms from left to right correspond to processes and procedures undertaken in the world to reduce uncertainty on the left to certainty on the right. There are agreements where the ideal solutions are remarkably efficient involving certainties in time (the lower triangle epistemological route) and surprisingly successful against all odds (the upper triangle ontological route). There are many isomorphisms to the risk homological structure where uncertainty to certainty are calibrated in terms of high risk to low risk, e.g., actuarial risk curve and the Shadow Banking system, and where positive solutions can be framed simply.
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<tr>
<td>5th Functors</td>
<td>Product from coproducts is the result of the general distributivity law so that all financial contracts can be added before and after a general default. &lt;br&gt;The general dynamic of default invariance comes from reducing uncertainty to certainty as a form of private bilateral settlement called Innovation ([I]) or as a matter of governmental trilateral settlement called Bailout ([B]). &lt;br&gt;The logical space of default invariance is ([I] \rightarrow [I] \rightarrow [B] \rightarrow [B]) where default is the morphism between each object. &lt;br&gt;The ring structure is a cyclic matrix with four phases: &lt;br&gt;(1) ([I] \rightarrow [I]) or ([i][I]), (2) ([i] \rightarrow [B]) or ([I][B]), (3) ([B] \rightarrow [B]) or ([B][B]) and (4) ([B] \rightarrow [I] \rightarrow [B]). This simplifies to: ([I][I] \rightarrow [I][B] \rightarrow [B][B] \rightarrow [B][I]).</td>
<td>The Great Cycle of Default Invariance is the macro-structure of the law and finance universe. &lt;br&gt;The Great Cycle of Default invariance is composed of four phases, with each phase composed of two states separated by default, with the form: ([I][I] \rightarrow [I][B] \rightarrow [B][B] \rightarrow [B][I]). The (\rightarrow) denotes a morphism. &lt;br&gt;Regulatory functors can be mapped between phases of the Great Cycle. &lt;br&gt;The Orderly Liquidation Authority of the Dodd-Frank Act is a regulatory functor from Phase 2 to Phase 3. OLA: ((I)[B] \rightarrow ([B][B]).) &lt;br&gt;Whistleblower incentives and protection is a regulatory functor from Phase 3 to Phase 1. WIP: ([B][B] \rightarrow [B][I].) &lt;br&gt;Unintended regulatory circulatory subsystems are 2-category pathways, e.g., OLA (\rightarrow) WIP in Phase 1 and Phase 2 tends to create a swift innovation to the bailout cyclic structure that emphasises a quick return to market conditions. But a OLA (\rightarrow) WIP in Phase 3 and Phase 1 tends to create a bailout-innovation-bailout-subsystem that tends to emphasise being stuck in government takeovers, expropriations and asset-bubble making.</td>
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Box 9.1  **Fundamental propositions of the Five Findings**

1  Commuting square
   - Commuting square $\rightarrow$ Financial contract
   - The commuting square is isomorphic to the contingent financial contract.
   - The contingent financial contract is the fundamental unit of the law and finance universe.

2  Commuting triangle
   - Commuting triangle $\rightarrow$ Epistemological and ontological routes to the annihilation of uncertainty.
   - $m$-morphism is the diagonal that arbitrages low- and high-risk derivatives contracts.

3  Shared codomain
   - Three approximations of financial reality are determined by shared codomain: $P$, $\sim P$, $P \& \sim P$.
   - $P$ implies single commuting square $\Rightarrow$ First Approximation of Law and Finance Universe.
   - $\sim P$ implies chain complex with Markets, Judicial and Political Systems to settle uncertainty into certainty $\Rightarrow$ Second Approximation is Risk Homological Chain Complex.
   - $P \& \sim P$ as coproduct assumes distributivity which implies a ring structure and for our purposes implies a cyclic matrix called the Great Cycle of Default Invariance (see Finding 5 for details).
   - Illustration of First and Second Approximation: *de jure* and *de facto* credit rating agencies correspond to First and Second Approximations, respectively.

4  Chain Complex
   - Second Approximation of Law and Finance $\rightarrow$ risk homological chain complex of Market, Judicial and Political systems, i.e., essentially, linear composite structure.
   - Essential zero mapping: $0 \_{\sim}:X \rightarrow 1 \rightarrow 0 \rightarrow Y$.
   - RHCC is isomorphic to Shadow Banking system.
   - RHCC is isomorphic to actuarial risk curve.
   - RHCC is isomorphic to risk symmetries framework and therefore, isomorphic to modern finance theory and prospect theory of behavioural finance.
   - Resolution of uncertainty to certainty is a composite from infinite-contingency to uncertainty annihilation.

(continued)
5 Functors

- Third Approximation of Law and Finance Product Universe \( \rightarrow \) Great Cycle of Default Invariance, i.e., essentially, cyclic group structure.
- Uncertainty and default are for purposes of law and finance essentially equivalent in that it is in principle impossible to know when or how default may be cured except by way of the judicial system, and if not by way of the judicial system then by way of political bailout.\(^{23}\)
- Financial regulations may be mapped onto the Great Cycle forming commuting triangles.
- Regulatory functors may interact as higher \( n \)-categories creating unintended circulatory subsystems within the Great Cycle of Default Invariance.

We explain the meanings of each of the five findings in turn. To understand their meanings means having a clear idea of the syntactic diagrams and their interpretations in the law and finance discourse. In brief, our analysis is a matter of defining, comparing, translating back and forth between NCT diagrams and law and finance discourse.

Table 9.2 NCT gadgets and applications to law and finance practice

<table>
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<tr>
<th>NCT syntactic structures</th>
<th>Applications to the law and finance universe</th>
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<td>Commuting square</td>
<td>Financial contract</td>
</tr>
<tr>
<td>Commuting triangles</td>
<td>Financial derivatives: low risk and high risk</td>
</tr>
<tr>
<td>Composites</td>
<td>Epistemological route of legal certainty versus ontological route of legal liability</td>
</tr>
<tr>
<td>( m )-morphism</td>
<td>Synthetics, arbitrage, law of one price, regulatory arbitrage, political arbitrage</td>
</tr>
<tr>
<td>Shared codomain:</td>
<td>Payment; Non-Payment; \textit{de jure} versus \textit{de facto} credit rating agencies</td>
</tr>
<tr>
<td>terminal object</td>
<td></td>
</tr>
<tr>
<td>Coproducts to product isomorphism ( \rightarrow ) ring and cyclic matrix</td>
<td>Payment and Non-Payment; macro-states of financial innovation versus government bailout; phase changes in market behaviours</td>
</tr>
<tr>
<td>Chain complex</td>
<td>Market risk; legal risk; political risk. Shadow Banking system; supply chain systems for transformation of maturity, credit and liquidity</td>
</tr>
<tr>
<td>Functors</td>
<td>Financial regulations; orderly liquidation authority; systemic risk; whistleblower incentives and protection; Basel III</td>
</tr>
</tbody>
</table>
Finding 1: The commuting square is the fundamental structure of all financial contracts

The syntactic structure of the law and finance universe at the nuclear level is the commuting square and thus, the commuting square is the structure of each and every financial contract. Trivially, a financial contract is isomorphic to the commuting square.\textsuperscript{24} Sharpe borrowed from Arrow and Debreu the concept of “contingent claims”\textsuperscript{25} to deal with risks which could be defined in terms of two time periods with a transfer of a commodity at the second time period (Sharpe, 1993, p. 2). These three Nobel Laureates in Economics independently set out the foundations of financial economics. It is obvious by inspection that the commuting square underlies the structure of contingent contracts and a formal rendering of contingent contracts in NCT terms and notation is the financial contract. For a definitional diagram of the commuting square, see Figure 9.8. For a definitional diagram of the financial contract, see Figure 9.9. For the obvious comparison of the commuting square and the financial contract, see Figure 9.10. And, for an elementary “proof” of the universal category as a financial contract, see Figure 9.11. The financial contract is the fundamental unit of the law and finance universe of the Default Invariance Theory.\textsuperscript{27} Just as Sharpe saw that there are a large number of implications from the Arrow-Debreu concept of contingent contract, so too there are an even larger number of implications from the concept of a financial contract as a commuting square. Put simply, once we show how the financial contract is a fundamental unit of law and finance in category theory terms, we get the rest of category theory for free. This is a slogan worth remembering, since it encapsulates the surprising non-trivial results that come from the unifying power of category theory. We will discuss the implications of the financial contracts as a commuting square in terms of three approximations of law and finance reality (see Finding 3 below), we can see that the commuting square of Figure 9.7 and the financial contract of Figure 9.8 are equivalent in the sense of being isomorphic up to the level of uniqueness as per Figure 9.9. In Figure 9.10, we have essentially the same argument of

\[
\begin{array}{c}
\text{NCT Syntactic Structure} \\
\text{Commuting Square} \\
\begin{array}{c}
A \\
\downarrow f \\
B \\
\downarrow g \\
C \\
\downarrow h \\
D \\
\downarrow i \\
A \\
\end{array}
\end{array}
\]

Assignments
- Morphisms: $f, g, h$ and $i$
- Objects: $A, B, C$ and $D$
- Shared Codomain: $C$
- Commutation: $gf = hi$

Figure 9.7 Commuting square
A naïve category theory of law and finance

Figure 9.8, where the financial contract which is “proven” to be the object of NCT is also the object of category theory in general.

Finding 2: Commuting triangles imply high-risk and low-risk financial derivatives

At the sub-nuclear level, so to speak, the commuting square is composed of two commuting triangles, and the triangles are isomorphic to each other.\(^{29}\) We distinguish the inner triangles from the outer routes. The inner triangles characterise the *de jure* aspects of the financial contract while the outer routes are the *de facto* actions taken by individuals.\(^{29}\) See Figure 9.11.

![Diagram](image)

*Figure 9.8 Financial contract*

![Diagram](image)

*Figure 9.9 Comparison between commuting square and financial contract*
A naïve category theory of law and finance

One way to interpret the triangles is to conceive of them as corresponding to two types of financial derivatives contracts, one with zero risk and legal certainty (i.e., $m = hi$), and the other with all manner of contingent risks and legal uncertainties (i.e., $m = gf$). See Figure 9.12.
Applied to the performance of financial contracts, the two commuting triangles divide the legal and financial universe into two different parts, one representing the epistemological route of non-probabilistic orderly determination (i.e., \( e = hi \)), and the other, representing the ontological route of maximal reduction of disorder (i.e., \( o = gf \)). Note how in the diagrammatic form in Figure 9.13, each of the two internal triangles elegantly corresponds to each of the two external routes. See Figure 9.13.

The complete description in words of the various relationships between the commuting square, the internal commuting triangles and the external routes, would be rather convoluted since our “natural English” does not flow from the grammar of the syntactic diagrams. Although we may be precise, we fall into the danger of sounding like gibberish. For example, we might say, “since both commuting triangles achieve the same result as the commuting square, said commuting triangles suggest two extreme types of financial derivative contracts, one type maximising epistemological certainty as in perfect information exchange, i.e., low or no risk de jure for some value of perfect information exchange and the other, minimising ontological uncertainty of information asymmetry, i.e., high risk de facto exchange and settlement of all risks.” Or, we can just inspect the diagrams in Figure 9.14, tracing the morphisms with our eyes and finger, and come to the same fairly obvious conclusions.
**Figure 9.13** Inner triangles as financial derivatives

**Figure 9.14** Commuting square → High-risk and low-risk financial derivatives contracts
Finding 3: Codomains determine approximations of legal and financial reality

Three shared codomains determine the Three Approximations

From the commuting square, we have three different types of shared codomains that determine three different types of approximations of legal and financial reality. The shared codomains are a terminal object “Pay”, another terminal object that turns into an initial object “Not-Pay”, and coproducts “Pay and Not-Pay”. See Figure 9.15. These three logical possibilities appear genuinely trivial but the structures which each implies are surprisingly significant, and form the broad context or environments of decision making in the law and finance universe.

We illustrate the meaning of the shared codomain as Three Approximations of Legal and Financial Reality by looking at each of their structural implications in terms of a macro-mapping. See Figure 9.16.

In brief, we can define the features of each approximation by taking the structure of each shared codomain to its logical conclusion.

First Approximation → Terminal object

In the First Approximation, since the shared codomain is the terminal object “Pay”, the implied structure is simply isomorphic to the commuting square corresponding to the ADSM. Another way of saying this is that the $t_0$-Agreement completes itself within the Market System. In this instance, the commuting square is *idempotent* (a structure referring to itself).
This is intuitively important because an idempotent usually indicates a bare structure that is elementary and combinable with itself. For example, the idempotent of the Category of Sets is the empty set and empty sets define the bare units that compose the units of natural numbers which are infinite. For our purposes, the financial contract as a commuting square serves as the idempotent for many other structures in the law and finance discourse but is completely unchanging.

**Second Approximation → Linear chain complex**

The Second Approximation whose shared codomain is “Not-Pay” initiates and dovetails into a linear chain complex where the “Not-Pay” shared codomain of the Market System is also the domain of the “litigation morphism”, which aims at the codomain of judgment in the adjacent Judicial System. In legal terms, non-payment at maturity calls a breach of contract that gives simultaneous rise to a right of action in a court of law for recovery of the unpaid amount due under the contract. In NCT terms, the “Not-Pay” shared codomain implies a more complex structure (i.e., higher dimension) than the “Pay” shared codomain. Intuitively (but not accurately!), the first approximation suggests a point, while the second approximation implies a line. The important structural concept to keep in mind is that to the extreme left of the chain complex is infinity contingency and to the extreme right is a zero, which effectively annihilates...
all uncertainty. Each of the adjacent commuting squares in a sense gives the participants in a financial agreement the opportunity to settle their differences either in a court of law, or where that fails, by some form of political action and so on in different types of decision-making fora. Being cognizant of the obvious essential difference between a first and second approximation gives a sense of the distinction between the \textit{de jure} (naively expected or interpreted under a very literal reading of the law and regulations) and the \textit{de facto} (how the law and regulations are actually used or abused for the interests of particular parties). We will illustrate the difference in law and finance practice between the First and Second Approximations by distinguishing \textit{de jure} from \textit{de facto} credit rating agencies. We call the structure implied by the “Not-Pay” shared codomain, the \textit{risk homological structure}. An important point of the risk homological structure is that it indicates how and why the risk of default becomes an object of contemplation for legal and financial practitioners. See Finding 4 for more details.

\textit{Third Approximation \rightarrow Cyclic matrix}

The Third Approximation whose shared codomain is “Pay and Not-Pay” is a coproduct. Making use of a simple Distributivity Law, which logically connects product to coproduct, we get a ring structure for free\textsuperscript{35} That is, addition and multiplication are operations of a ring structure. Since coproduct implies product, product and coproduct are directly linked via distributivity, we obtain a symmetric cyclic matrix structure, which we call the “Great Cycle of Default Invariance” (see Finding 5).

It is important to note that the Three Approximations are not just more “complex” in some vague sense,\textsuperscript{36} but are indeed, of different levels of well-defined structural complexity because of their determinable universal structures. As a mnemonic, the First Approximation implies the same point, the Second implies linearity, and the Third, a circle. Of course, these approximations do not literally correspond to a point, line and circle, but their qualitative different topological properties (their structure, in other words) make them \textit{approximately similar to the vast qualitative differences in form and dimension}, and these approximations as form precede any sense of quantification or calculation.

\textit{Applications of first and second approximations to credit rating agencies}

By applying the definitions of the first and second approximations to the concept of credit rating agencies, we can see their respective structural implications. In general, we can think of the function of credit rating agencies as establishing and preserving information asymmetry between the issuer and investor in the Financial Market System. The \textit{de jure} credit
rating agency business model is to charge a certain price for creating the spread of information between the issuer and investor. If this information spread were to close, the de jure credit rating agency would have nothing to sell. The genius of the credit rating agency model is not that it has any new, important, or significant information to offer, but rather it induces a state of lazy dependency and lulls unwary investors away from their natural homework which should give them their specialist competitive advantage.\textsuperscript{37} It is interesting to note that this sort of “being lulled into taking more risk” than one would normally think is rational for one’s survival is indicative of infection by a parasite that lowers the guard of the risk-taking creature, and simultaneously, strengthens the links in the survival of the larger ecology. The analogy to credit rating agencies is that by allowing investors to lower their guard, the investors were able to take on more risk, and thus, the credit-rating-to-investor morphism encouraged more risk taking of the issuer-to-investor morphism. One of the unintended consequences of this “separation function” of the de jure credit rating agencies pre-Credit Crisis were thought to be accurate, despite obvious conflicts of interest and the moral hazard to maintain an information asymmetry spread. However, the general public was disabused of this presumption when credit rating agencies succumbed to catastrophic re-ratings that indicated even to laymen that credit ratings are of no statistical value. For example, during the credit crisis, S&P downgraded 9,000 ratings in the second half of 2007 (Levin (ed.), 2011, pp. 263–7),\textsuperscript{38} and on one day in January 2008, S&P downgraded 6,300 real estate mortgage-backed securities (RMBS) and 1,900 collateralised debt obligations (CDOs) (ibid., p. 246). To give one a sense of proportions, prior to these re-rating catastrophes, it was generally repeated in textbooks in finance that the default probabilities of AAA investment grade bonds carried a one in ten-thousandth default probability, that is, that one AAA bond in 10,000 would suffer a default in one year. Just after the credit ratings debacle in September 2009, I recall chairing a risk management conference in New York sponsored by Goldman Sachs and PRIMA, where the risk managers of major funds were openly saying that their policies declared any AAA bond should be treated with 100% default probability!

Two types of credit rating agencies: de jure and de facto

We make a key inference from the first and second approximations, that is, that there are at least two different types of credit rating agencies in reality (i.e., whether we like it or not), the de jure and the de facto:

a De jure credit rating agency – based on the First Approximation pure pay model wherein the only relevant and material question is whether the issuing legal entity in question will make good its financial obligation (here the suggestive notation is $P = \text{AAA}$), and
b De facto credit rating agency – based on the Second Approximation which reverses the sign of the terminal object from “Pay” to “Not-Pay” and therefore warns away potential investors from certain danger (here the suggestive notation is \( \neg P = XXX \)). See Figure 9.17.

The structure of the de jure investment rating “accredits” the bond in question and therefore, tends to encourage investors to buy the bond which in turn results in concentration bias, while the structure of de facto rating tends to signal danger, warning off investors, and is evidenced by reversing the meanings of credit ratings found in (a). From a legal perspective, the concept of accreditation comes within the concept of Freedom of Speech, since it is merely expressing an opinion. This First Amendment defence can hardly be justified as accruing privilege when plainly the information provided by the credit rating agency is wrong because of negligence, intentional fraud, moral hazard and where the agency purports any form of statistical accuracy or correlation to long-term credit-worthiness.

Only two extreme results matter

Collapse of credit rating gradations to bifurcation

A second more speculative inference from the syntactics of the First and Second Approximation is that finer gradations of credit ratings may actually be completely useless in times of general default, since the only distinction worth keeping in mind during those times is that which comes from the partition between “Pay” (e.g., investment grade) and “Not-Pay” (default). To put it bluntly, nothing in the analytical structures of the

![Diagram of De Jure Credit Rating Agency: AAA → Buy and De Facto “Credit Rating Agency”: XXX → Don’t Buy](image-url)
First and Second Approximations indicate any method towards a more nuanced quantification scheme! It may come as a shock or surprise to market practitioners that credit ratings have very little or no basis in enabling any quantification scheme at all except the digital. Even if there are thousands or millions of so-called “statistical analyses” based on de jure credit ratings, they are quite impotent to tell practitioners anything at all that may affect their decision making except to confirm “the asymmetric information spread” that the investor has in relation to the issuer. The simplest way to overcome this information asymmetry is for the investor to merely ask the issuer relevant and material questions and for the issuer to answer such questions honestly. This result has obvious repercussions as to how financial regulations should be framed. It appears the most direct route for financial regulations to realise the dream of systemic risk reduction is to first be aware that a Third Approximation is the most “realistic”, and that therefore, caveat emptor, though ancient, is the best solution to information asymmetry and moral hazard problems. That is, if the investor were simply to be made liable for “bad investment decisions” ex ante rather than make the issuer responsible to second-guess what could move the investor to buy or sell, then a lot of spurious ontological nonsense would disappear from prospectuses. Since investors can never know whether an issuer will pay or not pay, all that we can hope to do with any modicum of reliability is hold the seller liable only for answers to specific questions posed by the investor which the issuer failed to answer honestly.

Returning to the failure of credit rating agencies during the credit crisis, we would normally call such failures in the market as an example of the manifestation of liquidity risk. Although vaguely defined by regulations such as Basel III, liquidity risk is usually associated with the failure of either a buyer or seller to transact in a market, and it can also be characterised as the magnification or amplification of any type of risk that leads to failure of payment. The suddenness in down-ratings was thought generally as a problem of de facto insolvency and illiquidity. However, a more important message for practitioners especially for asset managers is that these re-ratings completely discredited any finer distinctions purported by the agencies. Thus, one of the important uses of NCT of law and finance is that it should provide us with an enhanced realistic appraisal of financial contracts not only within the market sphere but also in the regulatory-judicial and political spheres.

Third Approximation results in cyclic structure

Bifurcation implies cyclic phases

The third type of structure for the shared codomain is the coproduct comprising $P$ and $\sim P$. We will have much to say about this structure under Findings 4 and 5, but for now, we state without further qualification that
the Third Approximation implies a generator where default is preserved in four discrete phases composed of states of financial innovation and bailout bound by default. A generator is simply a mechanism for replication or repeating the same pattern over and over.\textsuperscript{44} This is the $S^N$ exponentiation structure where the system states repeat as in $S * S * S * \ldots S$. We know in theory that the coproduct $(P + \sim P)$ comes for a deeper structure called “distributivity”. This is not just a matter of quantification, but literally, about structure. See Figure 9.18. For our purposes, the importance of distributivity is that it implies: (a) a matrix calculation where multiplication is directly connected to addition, and vice versa (see Figure 9.19: Product to coproduct isomorphism); and (b) where multiplication and addition apply, we have a ring or cyclic structure implied.\textsuperscript{45} See Figure 9.20.

Having sketched the implications of the three types of shared codomains as Three Approximations of Law and Finance Reality, we examine in more detail the Risk Homological Structure. We shall see that given its linearity from uncertainty to certainty that it has isomorphisms to major models that call for risk reduction.

**Finding 4: Risk homology**

The Second Approximation implies a chain complex that we call the Risk Homological Structure. See Figure 9.21.

In order to “read” this map, a few things need to be kept in mind. First, it is important to note that this is the mapping from a $t_0$-Agreement. Second,
that the object of contemplation, as it were, is default and the avoidance thereof. Third, the homological structure is simply a matrix composed of linear commuting squares with a specific value for the shared codomain of each square. Technically, in homology, our risk homological structure is similar to a $\text{Ch}(\text{Mod-R})$, which is a fundamental and generic homological structure, where the terminal object has a value of 0 (see Simmons,
2011, pp. 28–30). More specifically, we limit our model to a single chain complex with four adjacent commuting squares corresponding to the Markets, Judicial, Political and Communications Systems from left to right, respectively, and where the shared codomain of each square may be determined by 0 or 1, and where the final terminal object is on the extreme right is determined by 0. Thus, we can interpret this chain complex as a risk homological structure since for each financial contract where there is an uncertainty of payment in the Markets System; if such uncertainty is not resolved to certainty (that is, paid in full) then such uncertainty will be the basis for entering the Judicial System when a full and final judgment could resolve the uncertainty of payment into a certainty of judgment. If the final judgment is rejected, then such rejection of the Judicial System means that the original uncertainty continues into the Political System until a political decision is reached. And this same process is reiterated in the Communications System where the final value is 0, because of exhaustion of matter falling into the typicality of the universe.

It is important to note that the Janus-faced characteristic of each of the commuting squares in the risk homological structure. Just as we have shown that the commuting square can be divided into two commuting triangles, one representing low-risk and the other high-risk, so too each of the \(m\)-morphisms divide the commuting rectangles into low-risk and high-risk triangles.

As a practical matter, imagine structuring a financial contract. The structurer must anticipate default and how this may be resolved at the levels of the Market, Judicial, Political and Communications Systems. The important attribute that the Second Approximation shares with the First Approximation is the personal-to-dyadic relationship of this space. In the First Approximation, all contracts settle so the object contemplation is simply the explicit terms of the agreement. But in the Second Approximation, the implied terms come to the fore and greatly matter. For example, to the extent that the modern Judicial System leans towards assuming that the parties to the transaction intended a legally binding contract,
and therefore, can hear evidence of actions and behaviours of the parties outside of the explicit terms of the transaction and indeed, where this is not sufficient, then the courts can look to the custom of the industry to fill in whatever gaps are found in the alleged contract. These are matters for the courts to consider. And if this particular transaction results in a default that has political repercussions, then legislators and politicians may be involved in resolving the uncertainty caused by the default. The political decision concerning the original uncertainty may be a grand act such as a *de facto* default of a sovereign or a change in currency that sweeps up the original contract in its wake, or, it can be transferred into another general social sphere of exchange, but eventually, the initial financial contract is settled at the limit as a matter of exhaustion in the Communications System.\(^{48}\) Simply put, parties to the original contract seek resolution of uncertainty to certainty in each system of exchange and, if not resolved, moves left to right down the chain complex until it is finally resolved as an elimination of uncertainty. It is important to recognise that while we are speaking here of “systems of exchange” that the relevant unit of observation is the interaction between individuals. The question of how this system of individual financial contracts links up to larger and larger groups is non-trivial. In the law and finance literature, this problem has many faces with various labels, such as, “systemic risk”, “contagion risk”, “black swan effect”, “shortfall risk”, “drawdown”, and many others. These various labels or forms of risk may be thought of as different morphisms to the same generalised object.\(^{49}\)

First, we would need to sort\(^{50}\) the types of financial contracts into *generalised objects*, to show commonality and even more importantly, a specification by way of indexing or parameterisation (Lawvere and Rosebrugh, 2003, p. 14), so that the specific features of the contracts in question correspond and fall within the criteria set out by the generalised object. However, this would not be enough to prove the existence of the identity and uniqueness of the contracts in the homological structure. For this, we would have to show that for each and every morphism and object in the homological structure there is a functor\(^{51}\) to some structure and vice versa. Fortunately, this is already done for us in terms of a generalised structure called a “chain complex” that resolves to the value of 0. Explicitly, our risk homological risk chain complex has the feature of resolving the initial contractual uncertainty at every terminal object point of each commuting square with a value of 0, or by continuation of uncertainty with a value of 1. Figure 9.22 compares the structures of the First Approximation and the Second Approximation.

A handy way to remember the essential structure of the risk homological structure is to think that it is a vast transformation from infinite uncertainty to a specification of nothing at all, that is, \(\infty \rightarrow 0\). This composite is captured more precisely in category theory by what is called the “zero mapping” or “zero maps”. See Figure 9.23. The concept of the
Zero map is very useful whenever we wish to make comparisons between some form of risk transformation process and homological chain complex. Essentially, if we can show that the transformation process is isomorphic to the zero map, then we have a structure that is also isomorphic to the risk homological chain complex (RHCC).

**m**₁-to **m**ₙ: Morphisms as ideal solutions to Markets, Judicial, Political and Communications Systems

The risk homological structure directly implies that there are **m**-morphisms, which cut diagonally across from the **t**₁₀-Agreement to the shared codomains of the Markets, Judicial, Political and Communications Systems. If we apply what we learned about **m**₁-morphisms in the Markets System.
more generally to the other commuting squares, we might say that the \( m \)-morphism is an ideal contemplation wherein the lower-risk commuting triangle is equivalent in result to the higher-risk commuting triangle. In legal-financial terms, we would say that the \( m \)-morphism is “arbitraging” the lower-risk and higher-risk derivatives contracts in the Markets System. We can also think of the \( m \)-morphism as the act of contemplation which one needs to undertake in order to achieve whatever result is required in the shared codomain. That is, in order to achieve the result in the shared codomain, the action of the \( m \)-morphism is in a sense balancing the time-claim certainties with the world-action-uncertainties. Although, we would be hard pressed to find a correspondent vocabulary, it appears that there are ideals of efficient contemplation to achieve results in the Judicial, Political and Communications Systems, which correspond to \( m_2, m_3, m_4 \) respectively. More generally, \( m_n \)-morphism is entailed at the very edge of annihilating all uncertainty or replicating uncertainty ad infinitum. We give various examples of \( m \)-morphisms in risk homology in turn.

\( m_1 \)-Morphism

In Figure 9.24, we have the typical construct of the \( m_1 \)-morphism for financial derivatives in the Markets System, which we have already examined in detail above.

\( m_2 \)-Morphism

As an example of an \( m_2 \)-morphism, we might consider a derivatives structurer’s contemplation of bankruptcy avoidance.\(^5\) See Figure 9.25.

In many financial derivatives contracts, there are clauses that qualify the issuer as “bankruptcy remote”, so that the original issuer of the underlying collateral assets, i.e., the “originator”, is by contract and prospectus declared to no longer own the underlying assets, and bankruptcy of said originator should therefore not affect the legal rights of the bondholders.

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Figure 9.24 \( m_1 \)-Morphism for financial derivatives in the Markets System
This particular aspect has been declared no longer available for publicly issued asset-backed securities, where the issuer must retain ownership and liability not less than a certain percentage (normally, 5%) of the underlying and therefore, of the whole issue amount.\textsuperscript{53}

\textit{m}_3\text{-Morphism}

In Figure 9.26, we have the \textit{m}_3\text{-morphism in the Political System, which may be thought of as contemplations of financial derivatives for the purpose of achieving political objectives, such as reduction of prison populations, reduction of teenage pregnancies, or whatever socio-political objective may be on the government agenda and budget. While the technologies for derivatives have been focused on a rather narrow band of markets, mainly interest rates, currencies and commodities, there is nothing theoretically to prevent the same technologies to be applied to any measurable index.\textsuperscript{54} For example, reference to temperature index and linked to the price of energy, have enabled a market to develop for weather derivatives. The objectives of contemplation for reaching the shared codomain of the Political System are evident, and the \textit{m}_3\text{-morphism simply says that ideal \textit{t}_0\text{-Agreements may be created to reach such objectives. See Figure 9.26.}

Although we emphasised the socially positive and ethically responsible type of financial derivatives for the \textit{m}_3\text{-morphism in Figure 9.26, the risk homological structure also tells us that the ontological route (i.e., \textit{g}_3\text{g}_2\text{g}_1\text{f}) represents a very large risk of failure or continued uncertainty.

\textbf{The structuring of structuring – The \textit{m}-morphism as arbitrage across all spheres of exchange}

The risk homological structure has important implications on the way we conceive of the financial structuring process. At an abstract level, to conceive of derivatives is a matter of anticipating the effects of the shared
codomains of particular commuting squares. This act of imagination is the $m$-morphisms of the risk homological structure. One of the important structural features to keep in mind about $t_0$-Agreements is that the original raison d’être whether explicitly acknowledged or not (and usually, it is not) is that of efficiency or in the language of law and finance practitioners, regulatory arbitrage. The $m$-morphism that we saw in Finding 2 explicitly maps “arbitrage” in a category theory way. As per Finding 2, the $m$-morphism helps us see the two types of commuting triangles that correspond to no-or-low risk financial derivative and high-risk-or-never-ever-pay-back financial derivative. Our contemplation of the horizon of performance is $m_1: t_0 \to Market\ Closure$, $m_2: t_0 \to Judicial\ Closure$ and $m_3: t_0 \to Political\ Closure$. We look to and hope for a simple ADSM outcome in the first instance. But if ADSM is not available or does not eventuate, then we need to be prepared to pursue $m_2$. And if we cannot avail ourselves to $m_2$, then we must resolve to find a solution in $m_3$. Note that this form of contemplation is not what textbooks describe as financial derivatives, because in textbooks, the concern is only about reaching a calculation of value or price, not about actually accomplishing a completion of the actions which produce a legally binding and enforceable derivatives agreement. In other words, textbooks on finance rarely if ever teach us that while valuation and pricing may be calculated using formulae, the ultimate end of derivatives contracts is the same as in all matters of legally binding contracts, which is to achieve legal certainty, a form of ultimate risk reduction for all parties concerned.

What this notation means in the practice of law and finance is that at the point in time of structuring a financial derivative (at $t_0$, in other words), it is certainly the case that we think of financial products that give us efficiency in the Markets, Judicial and Political Systems. One might think of the structuring phase as the setting of an algorithm or tracking device. For example, with the bells and whistles of options, bonds and equity, we are able to structure the pay-off for specific events in the future. For example, consider the collateralised debt obligations (CDOs), credit default
swaps (CDSs) and repurchase agreements (Repos), three instruments of
the Shadow Banking markets. Each of these instruments allow otherwise
illiquid assets (i.e., untradable assets since they are not freely assign-
able) to be used as collateral for trade among bankers and corporations
in the highly active over-the-counter markets. As an example, one of
the principal legal and financial challenges of a CDO is that given the
enormous number of unique financial instruments composing a single CDO, how
could it overcome the potential costs of bankruptcy since multiple
instruments logically entail plethora of bankruptcy proceedings in case of
default? The solution was to create layers of capital with different levels of
priority in response to default. Thus, a default would “burn through” the
most risk-exposed form of capital; a bigger default would burn through
another layer, and so on, until there was no capital left. In this way, the
legal mechanism within the CDO made going to bankruptcy completely
useless from an economic standpoint since, if a CDO ever reached bank-
ruptcy, it would by operation of its contractual legal mechanism have no
collateral or assets to satisfy any judgment in bankruptcy. In this sense,
a CDO has an $m_2$-morphism that looks to obviate bankruptcy. Again,
financial textbooks rarely mention the reason for the enormous success of
some financial instruments over others because these successful products
provide solutions to problems that are not purely in the financial mar-
ket, that is, these products aim to solve problems that are of at least the
$m_2$-type, if not the $m_3$-type.

With regard to a CDS, one of the motivations of the structure is to allow
position taking on non-liquid assets to become tradable, resulting in the
parties to the transaction taking what amounts to equity-risk in transac-
tions involving bond instruments. Another way to say this is that CDSs
allow for credit-risk to be transformed into equity-risk. This risk trans-
formation is anticipated implicitly in the $t_0$-Agreement and triggered into
a legal actuality upon a declaration of default at $t_n$. The morphism $h$ in
the commuting square is a legal certainty, and it must be de facto a legal
certainty, otherwise, there could be a failure of the agreement within the
Markets System, and litigation would be required to quell the uncertainty.

Consider the Repo (repurchase agreements and reverse repurchase
agreements) that in practical terms aims to give near-infinite short-term
flexibility and arbitrarily large leverage to banks, which in turn are ironi-
cally presumed to be under strict regulatory leverage ratios, and thus, if
successful, achieve regulatory arbitrage. The de jure regulations in this
area appear to crucially misunderstand the very simple de facto purpose
of these financial instruments. Repos for short-term liquidity purposes
appear unobjectionable since those are exactly the little-to-no-risk-right-
commuting-triangles under Finding 2. But repos for long-term maturities
would entirely defeat the purpose of “safe” instruments, since long-
maturities with unregulated re-hypothecation is a formula for physical
shortfalls of collateral. Perhaps, a rendering of how particular financial
derivatives may be interpreted as $m_1$, $m_2$, or $m_3$-diagonals is no longer such a bad abstract idea! It would certainly improve the practice of risk management. By simply asking the issuer or user of a financial derivative what is their interpretation of the $m_1$, $m_2$, and $m_3$-morphisms, it becomes possible to understand what the possible effects (uses and abuses) are of the instrument in question to the Market, Judicial and Political spheres.

Another example on the possible practical use of risk homological analysis is in the notorious European bailouts of 2010–13. Europe had been embroiled in a series of “bailouts” involving Ireland, Portugal, Greece, Spain, Italy and Cyprus. These negotiations and re-negotiations between European peripheral countries and the Troika (the IMF, the EC and the ECB) can be considered as occurring within the Markets, Judicial and Political commuting squares of the risk homological chain complex (RHCC). And many of the so-called solutions are merely political announcements about laws or policies creating new institutions sometime soon. These political announcements at $h_3 \rightarrow h_3$ in the political sphere of the RHCC many times only generate a momentary cover-up of the structural unpaid debt at the Market System and Judicial System levels. More generally, from a practical legal and financial perspective, the structurer whether at the Troika level or at the individual financial derivatives level, needs to take into account risks that may eventuate across the RHCC, and it is in these sorts of contemplations that new solutions may be imagined. As we shall see, solutions to the US credit crisis was a matter of plugging gaps in the funding of the risk homological structure of the Shadow Banking system. This may appear to have resolved the problem from the perspective of the Second Approximation, but again, at the risk of getting ahead of our argument, there are major unintended consequences that we cannot see in the context of a RHCC, but are more likely to be made more apparent within the context of the Third Approximation.

Finally, although we have focused on particular personal-to-general structures of the RHCC (i.e., from $t_0$-Agreements through $m_1$, $m_2$, $m_3$, ..., $m_n$), we can use the RHCC as a whole to compare the assumptions of our analytic perspectives. This is interesting in the sense that this methodology may help alleviate our tendency to biases, especially, if we can recognise how the dimensions of our decision making are different or not from the RHCC. Where there is a strong similarity in terms of high-risk to low-risk dimension, then there may be an isomorphic invariant. We present a few of examples of what we mean by isomorphic invariants below.

**Risk homological chain complex isomorphisms**

At even a greater risk of abstract nonsense and out of a compulsion to complete the picture in terms of pure symmetry, we observe the risk homological structure is isomorphic to what is known as the *Shadow Banking supply chain* and the *actuarial risk curve*. These are extremely different areas
of practice and would not normally be thought of as even comparable in practice – one being in the dealer-to-dealer markets of credit instruments and the other, used as a risk quantification scheme for insurable risks. However, given their linkage to the risk homological structure, they suggest a deep unity in the risk management of financial derivatives and insurance contracts, and therefore, are of some fundamental importance to the practice of law and finance.

**Shadow Banking risk homological chain complex**

A major theoretical breakthrough in terms of the analysis of the financial crisis has come through the publication of the simple mapping of the Shadow Banking System provided by Pozsar and colleagues (2010, rev. 2012). In brief, Pozsar and colleagues have shown that we can conceive of all the credit intermediation and asset creation in the dealer-to-dealer market of the United States in terms of what I would call “supply chain”. In this supply chain, specialists compete to provide very efficient and competitive financial products that generally are aggregated with other products and are sold under different legal entities, moving generally towards higher credit quality with transformations of maturity, liquidity and credit. This mapping is captured by Pozsar and colleagues as a seven-step process of Shadow Banking, which can have as low as three steps and as high as eleven. The supply chain aspects can be clearly seen by examining a diagram provided by Pozsar and colleagues, which we have reproduced in Figure 9.27 (ibid.).

This Shadow Banking System can be translated into category theory terms as an isomorphism between the Assets Flows ($A$) and the Funding Flows ($F$). In other words, we can think of $A$ and $F$ as morphisms, that is, real-life processes where maturity, credit and liquidity are transformed, and identities are preserved. See Figure 9.28.

With the use of the zero map, it is also obvious that since the Shadow Banking System is a transformation of lower-quality to higher-quality assets that it is isomorphic to the risk homological chain complex (RHCC). See Figure 9.29.

We will return to the Shadow Banking risk homological structure when we examine the Default Invariance Cycle under Finding 5. By way of anticipation, we shall see that the major differences in Shadow Banking System pre-credit crisis and post-credit crisis has a “natural” explanation in the context of the Default Invariance Cycle which is the major model of the Third Approximation. More generally, we shall see that risk homology itself has different meanings in the context of the cyclic matrix. While the finding of an isomorphic-invariant structure of Shadow Banking and its differential structures under a Third Approximation are major points of this chapter, we also wish to point out that the risk homological structure is prevalent in different forms of risk assessment.
Figure 9.27 Shadow Banking

"Asset Flows"

1. Loan Origination
2. Credit, Maturity & Liquidity Transformation
3. Loan Warehousing
4. ABS Issuance
5. ABS Intermediate
6. ABS CDO Issue
7. Wholesale Funding

"Funding Flows"

- ABCP
- Repo
- CP
- ABCP, Repo
- $1 NAV
- "Funding Flows"
- "Asset Flows"
Figure 9.28  Shadow Banking Asset to funding isomorphism

Figure 9.29  Shadow Banking asset transformation and RHCC isomorphism
The trivial point is that graphic devices that have as a dimension low-risk to high-risk are isomorphic to the RHCC. For example, the actuarial risk curve used to determine expected, unexpected and exceptional risk of loss for corporate business risk and financial derivatives risk is isomorphic to the RHCC. See Figure 9.30.

At a general level, we can map a risk symmetries framework (RSF) where the x-axis is Perceived Risk and the y-axis is Definition. The RSF incorporates both the rational risk-return premise of Modern Finance Theory and the skewed irrationality of Prospect Theory and Behavioural Finance. In Figure 9.31, we see the RSF and RHCC are isomorphic.

Risk homological chain complexes appear to pop up everywhere in risk analysis and risk assessment and for this reason may at first seem trivial. However, because they are all isomorphic invariant, it shows that there is a deep underlying unity at least in the way in which we approach risk analysis in diverse fields, industries and professions. This unity indicates a monoid structure of the chain complexes themselves.

**Risk homology of the Shadow Banking System and the US Fed’s new purchase and lending programs of 2008–09**

Although we have already defined the Shadow Banking System in terms of the RHCC, we wish to show how the RHCC as a methodology might be

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**Figure 9.30** Isomorphism of the RHCC and the actuarial risk curve
brought to bear on finding solutions to credit crises. In summary, Pozsar and colleagues (2012) argue that the failure of certain asset-creation markets led to the credit crisis, and it was the US Fed’s new purchase and lending programmes that plugged up the funding gaps, and as the authors aptly put it, “the run on [the Shadow Banking System] was fully checked” (ibid., p. 25). The details of this thesis are vast, but fortunately, we can summarise the major Fed purchase and lending programmes as per Table 9.3, and show where each of these programmes fit in to plug the funding gap in the Shadow Banking System (see Figure 9.32).

**How the Fed fixed the credit crisis**

To illustrate how risk homology can be used as a matter of structural calculation, consider the US Shadow Banking System, its systemic collapse in 2007 and 2008, and the US Federal Reserve’s response in terms of its new purchase and lending programmes in 2008–09. This is an extremely complex set of facts for which many millions of words have been issued and which we shall now attempt to simplify into a few isomorphisms. First, consider Pozsar’s seven-step generic model of Shadow Banking (see Figure 9.27 above) as the basic asset-creation supply chain before the credit crisis. Second, we shall assume with Pozsar that the credit crisis was actually the failure of parts of this supply chain. For example, the real
estate mortgage-backed securities market failed and caused other parts of the supply chain to come under grave risk of complete stoppage. Third, to remedy this massive lack of private funding, the US Federal Reserve stepped in with its new purchase and lending programmes. See Table 9.3 for a summary of the various Federal Reserve programs.

Since each of these Fed lending or purchase funding programmes plugged a gap in the supply chain of the Shadow Banking System of the United States, we can think of the entirety of the risk homological chain structure being repaired and preserved through the Fed’s actions. These Federal plugs, so to speak, prevented the entirety of the Shadow Banking System from collapsing. So, again from the perspective of the Second Approximation of Legal and Financial Reality, we have a RHCC structure. This may sound all fine and good; however, as we shall see under the Third Approximation, the Fed’s actions indicate that the larger context in which the world of law and finance is operating is different. In a word, prior to the credit crisis, the Shadow Banking System was operating in the context where default was quickly remedied within the bounds of the Markets System. However, post-credit crisis, the Shadow Banking System being anchored and fed huge amounts of FRNs by the Federal Reserve is a supply chain that is structurally dependent on government support. So, we have the Shadow Banking System post-Federal purchase and lending programmes, with the US Fed taking over the function of the private funding facing the same risks of unbearable fluctuations. See Figure 9.32.

We can therefore think of the credit crisis and its solution in structural terms, that is, in terms of two isomorphisms, one between the risk homological structure and the Shadow Banking System, and the other between the Shadow Banking System and the US Fed’s new purchase and lending programmes. See Figure 9.33.

We can also see that the Fed’s new purchase and lending programs are isomorphic to the risk homological structure. So, we have a simple structural calculation of risk homology to shadow banking from shadow banking to Fed new purchase and lending program. One may be led to believe that because of these isomorphisms that the Shadow Banking System is substantially the same before and after the credit crisis. Yes and no. As we shall see in Finding 5, under the Third Approximation of the Law and Finance Universe, the government supported Shadow Banking System is part of an unintended circulatory system within the Default Invariance Cycle.

Finding 5: Functors

In Finding 5, we take the logical implication of the coproduct $P$ and $not-P$ as the Third Approximation of law and finance discourse. Recall the coproduct implies product through the law of distributivity, and together, addition and multiplication are the operations of a ring. The ring structure for
### Table 9.3 Summary of US Federal Reserve purchase and lending programmes, 2008–09

<table>
<thead>
<tr>
<th>Programme name</th>
<th>Operational start date</th>
<th>Description and purpose</th>
</tr>
</thead>
<tbody>
<tr>
<td>Term Auction Facility (TAF)</td>
<td>First auction: December 2007</td>
<td>Federal Reserve auctions “term funds” to depository institutions. All depository institutions that are eligible to borrow under the primary credit programme will be eligible to participate in TAF auctions. All advances must be fully collateralised. Each TAF auction will be for a fixed amount, with the term’s loan interest rate determined by the auction process (subject to a minimum).</td>
</tr>
<tr>
<td>Term Securities Lending Facility (TSLF)</td>
<td>March 2008</td>
<td>Weekly loan facility that promotes liquidity in Treasury and other collateral markets and thus fosters the functioning of financial markets more generally. Program offers Treasury securities held by the System Open Market Account (SOMA) for loans over a one-month term against other programme-eligible general collateral. Securities loans are awarded to primary dealers based on a competitive single-price auction.</td>
</tr>
<tr>
<td>Primary Dealer Credit Facility (PDCF)</td>
<td>March 2008</td>
<td>An overnight loan facility that provides funding to primary dealers, in exchange for a specified range of eligible collateral.</td>
</tr>
<tr>
<td>Asset-Backed Commercial Paper Money Market Mutual Fund Liquidity Facility (AMLF, or ABCP, MMFLF)</td>
<td>September 2008</td>
<td>Lending facility that provides funding to US depository institutions and bank holding companies to finance their purchases of high-quality asset-backed commercial paper (ABCP) from money-market mutual funds under certain conditions. Intended to assist money funds that hold such paper, in meeting demands for redemptions by investors and to foster liquidity in the ABCP market and money markets more generally.</td>
</tr>
</tbody>
</table>
Money Market Investor Funding Facility (MMIFF)  
November 2008 – didn’t become operational until 1 September 2009  
MMIFF (authorised by the Board under Section 13(3) of the Federal Reserve Act) supports a private sector initiative designed to provide liquidity to US money-market investors. New York Fed will provide senior secured funding to a series of special-purpose vehicles to facilitate an industry-supported private sector initiative to finance the purchase of commercial paper not backed by assets from participating money market funds.

Commercial Paper Funding Facility (CPFF)  
October 2008  
Created by the Fed to provide a liquidity backstop to US issuers of commercial paper. Federal Reserve Bank of New York finances the purchase of highly rated unsecured and asset-backed commercial paper from eligible issuers via eligible primary dealers. Money-market mutual funds and other investors had become reluctant to purchase CP, especially at longer-dated maturities. An increasingly high percentage of outstanding CP had to be refinanced each day, interest rates on longer-term CP increased significantly and volume declined. CPFF is intended to increase availability of term CP funding to issuers and to provide greater assurance to issuers and investors that firms will be able to roll over maturing CP.

Term Asset Back Securities Loan Facility (TALF)  
March 2009 – TALF financing also available for private-public partnerships to purchase toxic assets from banks.  
TALF is a funding facility that will help market participants by supporting new issuances of asset-backed securities (ABS) collateralised by student loans, auto loans, credit card loans and loans guaranteed by the Small Business Administration (SBA). Federal Reserve Bank of New York (FRBNY) will lend up to US$200 billion on a non-recourse basis to holders of certain AAA-rated ABS backed by newly and recently originated consumer and small business loans. FRBNY will lend an amount equal to the market value of the ABS less a haircut and will be secured at all times by the ABS. The US Treasury Department – under the Troubled Assets Relief Program (TARP) of the Emergency Economic Stabilization Act of 2008 – will provide US$20 billion of credit protection to the FRBNY in connection with the TALF.
Figure 9.32 The Fed plugs
Figure 9.33 Shadow Banking System funded by the Fed is isomorphic to the RHCC

our purposes serves as the basis for a mapping of the Third Approximation as a cyclic matrix wherein we have states of the world, [I] for financial innovation wherein contracts even in default settle between the private parties and [B] for bailout wherein contracts in default settle only through the support or takeover by government. In a word, the Third Approximation yields a default invariance cycle (see Figure 9.20 above) that can be represented in various ways but we shall prefer to represent it in terms of four phases A, B, C and D, with two states for each phase divided by default. See Figure 9.34 for default invariance cycle forms.

In Figure 9.34, the four phases appear rather arbitrary but in arguendo, are distinctions that are simply the law and finance distinctions taken to their logical conclusions. The characteristics of these four phases are briefly sketched in Figure 9.35.
Given the four phases in the default invariance cycle, which is the macro-structure implied by the $P$ and $\sim P$ shared codomain of the commuting square, we have an instance of the type of strategies which occurs in some of the more sophisticated parts of category theory. Note that the strategy here is to move from what we know about a local micro-structure (“the commuting square”) and linking it to a global macro-structure (“the default invariance cycle”). For purposes of the practical application to law and finance, we note that the Three Approximations give us structures
in which to contextualise our law and finance observations. Thus, in the First Approximation, we simply have single contracts settle. In the Second Approximation, we have contracts which may ramify out of the Markets System into the Judicial System and even further out. This means that as practitioners we need to focus on the $m_*$-morphisms, or contemplate on the points of termination in which the agreement in question can be liquidated or settled with finality in different spheres of exchange. And beyond the risk homological chain complex of the Second Approximation, we have cyclic matrix of the Third Approximation, which puts a completely different context again on the structures, which we had defined under the First and Second Approximations. In order to understand what we are dealing with in the Third Approximation, it is important that we have another gadget granted to us by NCT that allows for the comparison of more complex objects. This is the functor.

**Functor defined**

A functor is the first non-trivial gadget in category theory and used almost without blinking in NCT, that is, in the everyday practice of category theorists. It is defined as a morphism between two categories where each and every object and morphism is preserved, but not necessarily the same.\(^{58}\) It allows comparison between structures so long as those structures satisfy certain conditions, which are the conditions for being a category. Thus, one interesting non-trivial bit is that we can say that one functor is forgetful and the other free. The *forgetful functor* “forgets” some of the structure from the relatively-more-structured-source-category to the relatively-less-structured-target-category.\(^{59}\) And the *free functor* returns the missing bits of the lesser-structure to the greater-structure. As an example, say we have four chairs arranged in a square as the object $A$, and four dots arbitrarily arranged on a paper as object $B$. The four-chairs-in-a-square has more structure than the four dots on a page. So, $f: A \rightarrow B$ could be called the forgetful functor and $g: B \rightarrow A$ the free functor. We shall be using functors to develop complete syntactic diagrams of financial regulations.

**Cyclicity of the Third Approximation**

You may notice that in terms of planning and agreements, we would normally assume a risk homological framework. That is, the planning and agreement come in at the $t_0$-Agreement and the ideal aim is to reach a result at the shared codomain of one of the commuting squares in the risk homological structure. Thus, the bailout type solutions are $m_*$-morphisms targeting the shared codomain of the Political System. This type of thinking comes under what we call the Second Approximation of Law and Finance reality. The crucial point is that we plan within a context of linearity as per the Second Approximation. In contrast, under the Third Approximation,
the overall law and finance context is cyclic, and because of this cyclicity, that we believe we can return or control the law and finance universe.

Under the Second Approximation, we analysed the credit crisis of 2007–08 in terms of financial instruments that had intended uses, and regulations were simply lumped into the Judicial System and possibly the Political System. Under the Third Approximation, financial regulations become readily apparent since they become functors between phases of the Great Cycle of Default. So while the credit crisis as a phenomenon is the same in some objective sense, it is characterised differently depending on whether we view it as part of the Second or Third Approximation. We can conceive of the 2007–08 credit crisis as a default event, dividing the law and finance universe into a pre-default and post-default states of the world. See Figure 9.36.

Following Figure 9.36, we designate the Shadow Banking Market on the left-hand side of the default. It is actually suffused with private parties engaging in a supply chain money manufacture – i.e., assets flow versus funding contra-flow, where credit assets are aggregated, warehoused, standardised and enhanced in terms of transformations of credit, maturity and liquidity, which is mainly the Shadow Banking System. On the right-hand side of the event of default is the government using taxpayers’ funds or borrowing from its own treasury (as in the case of the US Fed) to purchase or lend funds in order to “stabilise” the Shadow Banking Market.

Figure 9.36 Pre-default versus post-default world
Stabilisation by the US Fed was really just a matter of the Fed supplanting the funding mechanisms normally supplied by private parties in a highly competitive supply chain. As we showed under Finding 4 above, the short of the long story of the credit crisis to credit solution was simply a “plugging up” of the funding gaps of the Shadow Banking System by the US Federal Reserve.

The credit crisis raised questions about the stability of the global financial system. But what is financial stability? Under the First Approximation, it means simply that we have ADSM contracts with no default. But this is extremely unlikely in reality. Likewise, under the Second Approximation, it is almost hope against hope that all contracts in default will ever be resolved in an annihilation of uncertainty. The most likely typical situation of the law and finance universe is to have coproducts “Pay” and “Not-Pay” operating throughout. See Figure 9.37 “First, Second and Third Approximations in Terms of a Coseparator”, and Figure 9.38 for an austerely simplification (or more technically, affine condensation).

**The macro-map of default invariance: regulatory functors change swarm behaviours**

As we have mentioned, the Third Approximation from the shared codomain $P$ and $\neg P$, which is a coproduct, implies a ring or cyclic matrix. This matrix, in brief, is conceived as two states of the world per phase
of the universe of law and finance. That is, each phase consists of a state pre-default and a state post-default. According to our model, each state can be either in financial innovation (that is, where contracts settle as is ADSM, and delay of payment $t_1$ is the manifestation of technical default, but is cured at $t_1 + a$ before $t_1$ which would require judgment by the court) or in bailout (that is, where default is generalised and where payment due is taken over by a government entity). The state of financial innovation is designated $[I]$ and the state of bailout is $[B]$. Between the two states are the functors of default, i.e., the forgetful functor from $[I]$ to $[B]$ and the free functor from $[B]$ to $[I]$. We presume a path dependency and therefore, there is a modal order $[I] \rightarrow [I] \rightarrow [B] \rightarrow [B]$ and the cycle is infinitely repeatable as in Figure 9.34.

We can use the four-phase map as the “context” for the macro-mapping not only for the “text” of regulations but also of all financial transactions, although they would be rather blurred since they would constitute hundreds of billions of transactions running along the morphisms $f, g, h$ and $i$.

The Third Approximation allows us to hypothesise that financial regulations are regulatory functors, since each phase is also a category. In a strong sense, financial regulations do more than merely “incentivise” actors to comply, since they set up patterns of transactions, which have a tendency to deform the normal trading patterns of the market, judicial and political landscape. Since these regions are so vast, our normal English vocabulary is simply inadequate to capture the “clouds of data” and the profound massive changes in swarming behaviour that regulations create. With the macro-mapping, we can at least make some hypotheses about what may happen if certain financial regulations were to take effect.
within the default invariance cycle. The Third Approximation could be used explicitly as a mapping of very large systems of financial markets, judicial and political interactions where financial innovation and government bailout and bail-in schemes are the predominate solutions to apparent continuous financial crises, such as, Europe in 2010–13. However, for purposes of illustrating the methodology of the Third Approximation, let us consider US examples of post-credit crisis regulatory control, i.e., the Orderly Liquidation Authority (OLA) and the Whistleblower Incentives Protection (WIP) under the Dodd-Frank Act of 2010. See Figure 9.39.

**OLA as a regulatory functor**

In the most direct effect sense, the OLA creates a short-cut between Phase 2 and Phase where 3 bailout-to-bailout is prevalent. This is not entirely implausible, since the primary action of the OLA is for the FDIC to take ownership and control of the good assets of any company posing a systemic risk to the US economy. The procedure is expedited within hours and the private owners are cursorily protected in their property rights by the lowest and risible standard of judicial review. Thus, OLA acts as some kind of monstrous catalyst or parasite that captures companies that “pose a systemic risk” and turns them into wards of the State.

**WIP as a regulatory functor**

At the other end of the default invariance cycle, the WIP creates a minimum entitlement of 10% of whatever recovery above US$1 million to be
paid to the informant who provides original information to any securities violation, and this may be filed and collected at either or both, the SEC or the CFTC. Given that recoveries in the securities litigation arena may reach into the billions of dollars (although not likely), the potential recoveries may induce the creation of a new litigation industry, complete with litigation investment funds and who knows, new financial instruments, that transmute expected future cash flows into net present value bonds. The potential of WIP to trigger new financial instruments means it fits well as a regulatory functor located on the road, so to speak, from Phase 4 to Phase 1.

**Shadow Banking as unintended circulatory systems**

If we presume that within the default invariance cycle regulatory functors interact, then certain cross-roads and bridges are created without any intention by the regulators or regulatees. Consider Figure 9.40: if you do a bit of diagram chasing and trace the composite $eyag$ and $bxdi$, you will see two circulatory systems: $eyag$ circulates between Phase 1 and 2 and $bxdi$ circulates between Phase 3 and 4. The Phase 2 and 1 circulation tends to keep the system in Financial Innovation and quickly comes out of Bailout while the Phase 3 and 4 circulation tends to keep the system in Bailout and quickly takes it out of Innovation. Let us call the Phase 1–2 circulatory system the “Virtuous Vault”, and the Phase 3–4 circulatory system, the “Government Vault”.

Within the Virtuous Vault, you may see a Shadow Banking System which is privately financed and has the least government interference or involvement. Within this context, the Shadow Banking System is a dealer-to-dealer market and thrives without government support. In contrast, within the Government Vault, the Shadow Banking System is directly supported by government asset purchases and lending programmes, and bailout-to-bailout means more and more government control of the asset manufacturing supply chain. In the Government Vault, quantitative easing (QE), open market transactions (OMT) and other types of direct central bank market activities are actually *de facto* government-sponsored derivatives programmes of the growing bailout to bailout $[B][B]$ phase of the Default Invariance Cycle. The exogenous saviour of the private markets of the Shadow Banking system of the Virtuous Vault is now an endogenous actor of the Shadow Banking System.

How plausible are these interactions? Many narratives could be told about how these might work out. However we interpret this map, if we use it, we will know approximately where we are in its cycle and this type of macro-mapping helps us understand the unintended and surprising interactions that are not anticipated by any intention of the rational agents. Just as a map should do: it provides us with information that we may use to guide our actions or ignore to our peril. See Figure 9.40.
Conclusion

This essay is a meditation on how the arrow leading to a point, circle, square and triangle are related to law and finance. These are not metaphors but ideals that take teleology seriously. Before we do certain kinds of calculations, we make use of ideals in order not to get lost, and to come to some firm conclusion about how to go about our business, and how to get to our destination. The discourse of law and finance is no different. We have used some extremely abstract concepts to come to conclusions about the universal properties of law and finance. These techniques are usually called “mapping” and come with precise definitions from category theory. We have only made a few infantile steps in abstracting the structure of law and finance, calling our tools “naïve category theory” in honour of the types of everyday well-worn gadgets used by category theorists. Our method has been to define the most prevalent phenomena of law and finance from both micro and macro perspectives. Between these extremes, we have persuaded ourselves that Three Approximations of law and finance reality come from the logical conclusions of three possibilities for the outcome of a contract at the time of payment. In this sense, the macro-structures are derived from the micro-structures, but we have also seen that the bridge between the micro and the macro can be travelled back and forth. From these Three Approximations, we have different settings in which law and finance phenomena can be observed and interpreted, and these observations and interpretations we call “five findings”. At the
nuclear and sub-nuclear level, the commuting square and commuting triangles, together with the \( m \)-morphism, give us the fundamental structures of the micro-level contemplations. These structures come into play in the Market, Judicial, Political and Communication Systems, and tell us that the bifurcation of low-risk and high-risk financial derivatives are constructed to target and mediate specific goals in these spheres of exchange. Law and finance matter because they naturally unfold from the approximations we have of their fundamental structure. The First Approximation completes only itself disappearing into a point, the Second defeats itself and requires resolution along a line from uncertainty to certainty in various forms of socially agreed ritualisations, and the Third creates a cyclic context in which regulatory texts and actions are finely frescoed. Although we would like to believe that of primary importance in the structure of law and finance is the reduction of uncertainty of contracts into the certainty of payment, judgment, political action or some other form of communication, the fact is that we are dipped and surrounded by uncertainty and default. We have also seen that a more realistic vision of the unknowability of the future as a logical complete coproduct of “Pay and Not-Pay” gives us a rather surprising cyclic matrix structure for the macro universe of law and finance. Each matrix is a phase composed of two states divided by default, and these matrices are linked by unbearable fluctuations from one phase to another. With the map of the Great Cycle of Default Invariance, financial regulations can be mapped as regulatory functors, which are determinates and sections between the four phases. This mapping also indicates that there are secondary effects not anticipated by the regulations themselves, which may be called unintended circulatory systems that derail our rationality and continuously seduce actors into the stickiness of certain regions. It needs to be emphasised that this essay has been largely limited to structural issues and not to quantification schemes. However, it is strongly contended that structural considerations are primary to any quantification scheme and are indeed the closest peel to the essential skin of the law and finance reality.

Notes

1. Teleology, one of the four causes of Aristotle’s canon, the others include substantive-essential, efficient and formal, has been relegated by the western scientific community since the Renaissance, mainly because it has been associated with arguments for the necessary existence of God. One might say that efficient cause with its sense of a “building block” or “materiality” informed most of the premises of scientific thought up to and including the twentieth century. From the mid-twentieth century, formal cause in the form of computation and virtuality has succeeded to the scientific throne. Teleology when it is denigrated is usually associated with some kind of anti-Darwinian biological determinism or Spencerian “human progress to” some Teilhard de Chardin “omega point of human evolution”. But this would be an abomination to the
A naïve category theory of law and finance

original Aristotelian teleology, which is simply a “well-defined purpose”. And for “well-defined”, we would need to refer to Aristotle’s Categories which looked to a coalescence between existent particular properties (“being in”) and universal attributes (“said of”), which is strangely similar to isomorphism up to the level of uniqueness found in modern category theory proper. And teleology as an explanatory vehicle, I believe, has never really disappeared in the way we think about how to resolve problems of uncertainty to certainty. Although category theorists are likely to be indifferent to this obvious association of their underlying tools to teleology, these same tools could be used to justify certain courses of human action and disciplines such as ethics and politics. In modernity, teleological interpretations play a vital role in understanding the principles of statutory interpretation of European Law. See Veil (2013, p. 57).

2 See, Aristotle’s Physics, Metaphysics and Ethics, et passim. Certain modern classicists argue that “teleology” as a philosophical doctrine did not appear until the 1800s but this assertion rather defeats the literal translation of the Greek and the pervasive use of “final causation” throughout Aristotle’s works. See, for example, Aristotle, Physics 2.8, 199b27–9 and Physics 2.5–6.

3 In category theory proper, we would say this “linking up” is captured by the abstract relation called “associativity”, whereby \( f(g*h) = (f*g)*h \). In the previous statement the “*” means an operation between two morphisms.

4 The square brackets here merely mean “the set of”, and we use them here to show an apposition.

5 Category theorists per se may baulk at this subjectivism, but if this sort of “explanation feels compelling”, then it should be noted that as a matter of record that the best introduction to category theory, Conceptual Mathematics (2nd edn) by F.W. Lawvere and S. Schanuel (2009) is presented in part with natural dialogues where secondary school students apparently are asked and answer questions about the meanings of diagrams. There is after all no inconsistency in interpretation so long as we maintain the isomorphisms between objects. This philosophical attitude is expressed in terms of isomorphisms with small categories. See Lawvere and Schanuel (2009, pp. 84–5).

6 See Lawvere and Rosebrugh (2004, p. 78), where span is simply a pair of arrows with a common domain.

7 A terminal object is defined axiomatically as \( f: A \rightarrow 1 \). Although this may appear rather trivial, since it means literally that the terminal object cannot distinguish any elements of \( A \), the dual of the morphism \( f: 1 \rightarrow A \) gives us an infinity of distinctions.

8 We shall define isomorphism technically later, but for now, it is important to note that it is the identity relationship between two objects. Conceptually, category theory gives us nuanced views of what it means to be equivalent and near-equivalence.

9 The concept of minds is recognised in the law as per a “meeting of minds” as a fundamental legal criterion for a valid contract. The “meeting of minds” has no physicality as such, but may be evidenced by physical evidence such as documentation. Philosophers of mind normally look to neuroscience or computation for analogies to their concerns, but the discourse of law and finance provides ample assertions of “minds” as separate in principle from any materiality.
The *Meno* illustrates how abstract thought requires someone who already knows the structure of the abstraction in question to pass it on to another. How the abstraction “jumps in” the mind of the one who knows is still a mystery. We might call this the question of the original isomorphism: “How did the first isomorphism come into being?” For a categorical explanation of the objective and subjective, see Lawvere and Schanuel (2009, pp. 84–5).

Technically, these various concepts for arbitrage would fit under the rubric of an isomorphism. It is tantalising to conceive of weak arbitrage as weak forms of equivalence, and more sophisticated (complex) category theory proper would be invoked in demonstrating that certain sorts of arbitrage are natural equivalences and adjunctions. Please note that I do not believe that this would be a worthwhile adventure unless and until some of the “lower level” theory promoted in this paper are susceptible to quantification.

By the way, this thought experiment of throwing all possible derivatives contracts not having a specific feature at a particular sort of action (netting) and coming up with just one specific universal feature (one value) is an application of the first non-trivial result of category theory called the “Yoneda Lemma”. But this result is not within the scope of this paper. However, the Yoneda Lemma does illustrate a type of “process of elimination” technique that could conceivably be applied to various situations requiring “risk management” technology. The Yoneda Lemma was attributed to Yoneda by Saunders Mac Lane (1978), in a note on p. 77.

For the rules regarding capital, see Basel Committee of Banking Supervision (2011). And for the rules regarding liquidity, see Basel Committee of Banking Supervision (2013).

We designate these $m$-morphisms by their distance in terms of adjacent commuting squares, i.e., $m_1$ for the financial Markets square, then $m_2$ for the adjacent Judicial square, then $m_3$ for the Political square and $m_n$ for a more general Communications square. The adjacent commuting squares form the chain complex we call “risk homology”. For a detailed discussion, see Finding 4.

Turning these “dreams” into reality is the function of the law and finance system, for in a broad sense the great transformations of trading in the financial markets are: liquidity, maturity and credit. See Finding 4 on Shadow Banking.

This just means that where we have a single element $a$ in an object $A$ and a single element $b$ in an object $B$, given arrows $f: A \to B$ and $g: B \to A$, and ensuring that the rules of associativity and identity apply, then the existence and unique identity of $a$ and $b$ are proved. Many times in ordinary law and finance language, we might say one concept is the same as another, and if we mean it literally, then it is an isomorphism up to uniqueness. For example, when we say a person is guilty of an offence, the particular person accused is isomorphic to the person who committed the crime so defined. If he or she is not, then it may be a case of mistaken identity.

With the concept of isomorphism in mind, it is possible to simply designate an object, say a rock, to represent another object, say a horse. Without knowing any abstract numbers, we can simply line up one rock per horse. We can reverse this process if we wished to trade our horses for a specific number of corn ears. This type of isomorphism underlies all quantification. See Lawvere and Schanuel (2009, pp. 40–41). The concept of settlement, for example, is simply an application of isomorphism. And in this sense, isomorphism underlies our concepts of arbitrage and fungibility.
As Bob Coecke states in his attempt to bring category theory formalism of quantum mechanics to kindergarteners (1): “It [category theory] concerns ‘doing quantum mechanics using only pictures of lines, squares, triangles and diamonds’ in Coecke (2005, p. 1).

19 Aristotle’s phrase of “actual actuality” is the state of something being itself in real time and space; it is the literal end-point of teleology. In category theory terms, an actual actuality is simply a composite of two morphisms say \( f: A \to B \) and \( g: B \to C \). The actual actuality is \( gf: A \to B \to C = A \to C \).

20 This vision of the information and communications system as a structure of certainty encoded within randomness was fully realised by Claude Shannon in his classic “A Mathematical Theory of Communication” (1948).

21 Legal theory has a long and distinguished history, which one might say reflects the theoretical fashions of the day. Hohfeld for example studied chemistry before law, and his theory of law was obviously very much influenced by the periodicity and pattern building of chemistry-like structures. His eight legal relations were arranged in terms of jural opposites and jural correlates, corresponding to a monadic analytic framework and a dyadic real-life framework. The dyadic arrangement is: rights versus duties, privilege versus no-right, power versus liability and immunity versus disability. These legal relations form distinctions which lawyers and judges use either consciously or subconsciously in their everyday jargon. Hohfeld’s genius was to see how these concepts were arranged in relation to each other, establishing a meta-pattern approach for legal discourse. See Hohfeld (1919, 2008, p. 6).

22 A finding might be thought of as what we see or observe using category theory lenses.

23 Estimating probabilities of default of course has been tried since Bachelier’s PhD thesis in 1900, but these exercises have only come to a negative result. Despite the theoretical applications of Markowitz (1952) of modern portfolio theory, and Sharpe (1964), Linter (1965), Treynor (1965) and Mossin (1966) on mean-variance equilibrium model of exchange, which together form the foundations for Modern Finance Theory (MFT), their statistical methods cannot be said to be accurate. The only statistic that appears to be true is that their probabilistic methods fail to predict anything in the financial markets with any modicum of confidence. This tremendous failure of normal statistical methods in finance does not appear to dissuade regulators from depending on such methods for justifying their financial regulations. See Candemir and Tanega, 2011; Bachelier, 1900, 2006. On why MFT has failed, see Mandelbrot and Hudson, 2006.

24 A more technical definition of isomorphism is that it is a relation that obtains where given the objects \( A \) and \( B \), and the morphisms \( f: A \to B \) and \( g: B \to A \), \( gf = 1_A \) and \( fg = 1_B \). Less trivially and I’d say much more significantly, the comparison between the commuting square and the financial contract can be seen in terms of functors. Thus, where the commuting square is a category \( A \) and the financial contract, a category \( B \), and \( f \) is a morphism from \( A \) to \( B \) \( [f: A \to B] \) and \( g \) is a morphism from \( B \) to \( A \) \( [g: B \to A] \), then \( f \) is a normal functor and is a forgetful functor. A functor maps each and every object and morphism from one diagram to another. A forgetful functor is a morphism that loses some of the structure in the mapping from \( B \) to \( A \). It’s important to note that technically, a functor must preserve the identity arrows and it must also preserve the composition of composable arrows, so we have covariant and contravariant
functors. But due to the limitations of space, we mention only how the categories A and B may look alike in an informal sense, but with the functors, there is an equivalence between the two structures which holds without stating that they are the same structure. In category theory proper, we would call the financial contract from the commuting square an instance of a universal mapping property. For a definition of functors, see Simmons (2011, pp. 73–5).

None of these three Nobel Laureates had any legal training and as economists, they tended to oversimplify the legal universe. Fortunately, the concept of contingent claim in the context of “sphere of exchange” instantly resolves into a contractual relationship. In other words, the logical context of “claim” which from a legal perspective is insufficient to produce a contract is for our purposes sufficient to be equivalent to a contract since a “claim” in the context of exchange is taken to be equivalent to a contract in its formation stage.

Figure 9.11 is intended for the sceptical category theorists who would like to see at least one tidy universalistic definition of financial contracts as a category per se.

This is the theory that default is everywhere and even in situations of full payment, it is still linked to the nature of how things are. It is more apparent in the Third Approximation of legal and financial reality, where the shared codomain is both “Pay” and “Not-Pay”.

The proof for this is very simple and follows from the fact that the commuting square and the two triangles share the same diagonal morphism, \( m \). So, \( gf = m = hi \).

Again, this shows the power of morphisms to distinguish different types of actions. The inner triangles share the morphism \( m \) and are therefore obviously different from the outer routes. Yet, the inner triangles and the outer routes share the same codomain.

For definitions and discussion of “parts” and “partitions” in category theory terms, see Lawvere and Rosebrugh (2004, pp. 20, 26, respectively).

John Holland in his classic (1975, 1992), *Adaptation in Natural and Artificial Systems* presents some code produced by some successful genetic algorithms, which he calls ugly gibberish. It’s “gibberish” and “ugly” only to the eye of the beholder. It is likely that after some practice “categorical gibberish” (“abstract nonsense”) makes good sense to mere humans.

For a technical definition of terminal object, see Lawvere and Rosebrugh (2004, p. 12).

Technically, an idempotent is a morphism from single object \( A \) back to \( A \). When a financial contract is completely performed, and when there is nothing left to do to fulfil any and all terms and conditions of the contract, then the contract in theory refers to itself in the sense that each action that was required to be done can be pointed at as having been done. A completely performed contract can be thought of as an “idempotent endomorphic” structure.

We might consider defining the commuting square as a monoid. That is, as a category with exactly one object where the commuting square as a whole is the object. See Lawvere and Schanuel (2009, p. 166). The reason for this sort of mapping is that we would then have the basic structure for a set-with-endomap such that the object is a functor from \( N \) to \( S \). That is, say the commuting square is \( S \) and the monoid is \( S^N \), then we’d have a category of dynamical systems called “discrete-time dynamical systems”. A discrete-time dynamical system is a functor from this monoid \( N \) to the category of sets, \( S \) (see Lawvere and
A naïve category theory of law and finance (Schanuel, 2009, p. 168). A monoid is simply a category where the morphism is to itself, and the concept is quite useful for unifying many apparently different types of discrete abstract gadgets.

Product to coproduct isomorphism under distributivity results in ring structure is a well-known result in abstract algebra. See Pinter (1982, 1990, pp. 169–70), where multiplication and addition are just two operations of a ring. We add a slight variation in that the area of distribution between the multiplicants and summands needs to be thought of in at least three dimensions in order to have a structure, which enables even the most basic quantification. The idea is that “Pay” and “Not-Pay” can generate the total number of payors and non-payors, and thus, the total number of legal entities within a financial Market System. In a state of financial innovation (I), where default occurs, non-payment is cured with other payors within the Market System. The total value of payment and non-payment should be equivalent pre- and post-default. The only difference occurs in terms the identity of the payors. However, with default, if government bailout occurs, additional costs surmount previous total obligations, leading to expansion in the form of asset-bubbles. A reduction of obligations occurs when such are retired at maturity or perpetuated due to low maintenance costs, but reversed once total service costs are greater than the total asset value. This dynamic expansion to contraction cycle is not anticipated by a product-to-coproduct isomorphism, but can be established with further structural gadgets that allow for automata and feedback. See Finding 5.

This sense of complexity in terms of differentiable forms or structures allows us to make comparisons between complex structures. Many times, legal theorists and practitioners are apt to use the words, “indefiniteness, ambiguity and vagueness” to indicate areas where the law cannot provide a remedy. Due to the constraints of space, we can only suggest without proof that such legal reasoning is susceptible to analysis under naïve category theory since such reasons for non-justiciability are indications of morphisms of non-action.

The idea that specialists in finance survive by their competitive advantage is one of the rationales for the differentiation of functions in the Shadow Banking Supply Chain.

For a record of the “mass downgrades”, see Levin and Senate Sub-Committee on Investigation United States (13 April 2011, pp. 263–7).

This is why high credit ratings tend to have initially an “accreditation effect” and support a sense of “over-confidence” bias. This effect has little to do with statistics and more to do with generating a belief structure within a highly communicative network.

For example, in the immediate wake of the financial crisis, the author was chairing a PRIMA conference on risk management held at Goldman Sachs and one of the speakers stated that his fund which specialised in repo intermediary funding had conducted a 6-month risk management study of AAA instruments and had concluded that as a matter of company risk policy that all AAA instruments would be considered as having 100% default probability! This is an example of the de facto reversal of the Second Approximation of financial reality where the terminal object is not-pay, that is, of a pure default model.

The US legal standard was stated in New York Times v. Sullivan, whereby opinions cannot be a basis for liability unless the plaintiffs can prove actual malice by the credit rating agencies. The credit rating agencies have also won
earlier cases based on the privilege of journalists and other shield laws. These are laws that provide a First Amendment defence on requests for discovery which seek information on the foundations for credit ratings. In June 2009, the California Public Employees Retirement System filed suit in San Francisco against Moody’s, Fitch, and Standard & Poor’s for negligent misrepresentation in their ratings of $1.3 billion in ‘structured investment vehicles’. For a synoptic analysis of the long line of legal cases, see Albert (2009).

Note how the concepts of “credit”, “accredit” and “discredit” have an etymological route in the Latin verb “credo”, which translates into “belief” and “faith”.

See Finding 5 below for Orderly Liquidation Authority and Whistleblower Incentives and Protection.


Of course, we do not mean that a single judgment within the regulatory-judicial system constitutes a rejection of the entire judicial system. We mean that where the judgment rendered amounts to a non-recognition of legal rights and duties embedded within the original intention of the parties at $t_0$, then this non-recognition could be considered unjust in a general sense. Note how the injustice of single legal cases may lead to the political action. Interestingly, one of major social critiques of the credit crisis is that it has not lead to more prosecutions and therefore, accusations of corruption and distrust of government actions are rampant. The hands of justice turn very much more slowly than the clearing and settlements in the market sphere. One might speculate, for example, that the legislated time limit for the terminal object in the judicial system is approximately the time limit built into the statute of limitations for particular legal actions. For example, nine major complaints against investment banks for securities fraud were filed by the agency in charge of the conservatorship of Fannie Mae and Freddie Mac, a few days before the statute of limitations were to run out on such claims. See, FHFA Filings in PLS Cases (2011). Clamouring for criminal prosecution can also be found within governments themselves. See, for example, Levin and Coburn (2011), where appendices are arranged as detailed indictments against credit rating agencies and specific investment banks. See Levin and Coburn (2011), “Case Study of Moody’s and Standard & Poor’s”, pp. 243–317; Levin and Coburn (2011), “Case Study of Goldman Sachs and Deutsche Bank”, p. 318 cf., Levin and Coburn (2011), “Running the CDO Machine: Case study of Deutsche Bank”, pp. 330–75 and Levin and Coburn (2011), “Failing to Manage Conflicts of Interests: Case Study of Goldman Sachs”, pp. 376–635.

The typicality of the universe is a typical location of the universe where no interaction is expected at all. See Deutsche (2009).

For example, claims on inactive bank accounts might be several generations old and long forgotten by those who have the legal right to be paid by the banks. Exhaustion is a feature of default.

The concept of the “generalised object” is useful indeed. We use it all the time, although perhaps unconsciously. Whenever we use the definite article “the”,
as in “the temperature of London in June”, we may make many temperature readings which corresponds to the domain and the morphism is “taking temperature readings of London in June” and the generalised element is “the temperature of London in June”.

Technically, this means that the morphisms would have a surjective nature, where each element of the codomain is covered by a morphism from the domain. Surjective morphisms assert the existence of elements in the codomain.

A functor is a morphism between two categories. In brief, a functor compares two categories using two morphisms: one morphism is from objects of one category to the objects of the other category, and the other morphism is from the morphisms of one category to the morphisms of the other category. However, this second morphism can be covariant (going in one direction), or contravariant (going in the opposite direction). For this chapter, unless otherwise indicated, we shall use the covariant version.

To understand how a legal structure actually (and not just theoretically) avoids bankruptcy requires quite a lot of sophisticated legal knowledge. To get a flavour of the complexities which a lawyer may have to resolve, see Lubben (2011).


Usually, the phenomenon in question must be observable and measurable in an objective sense. For example, the number of prisoners and the number of teenage pregnancies are countable and patterns may occur in a statistically significant way. This statistical pattern becomes an index, which may become the reference index to a basic financial derivative, such as a swap. The swap could be structured in a way to allow for increased efficiency in the allocation of public funds towards the reduction of teenage pregnancies. I would like to thank Professor Edmond Curtin, of the University of Westminster, School of Law, for pointing out this specific type of derivative and many other socially and ethically engineered financial products.

For example, in the notorious Abacus Prospectus, there were 90 separate real estate mortgage-backed securities in the portfolio of the CDO. See Abacus 2007-AC1, Ltd (2010).

Although in practice, the law of finance and banking is very different and indeed, “silied” from insurance law practice, at the theoretical level, there have been major attempts to stitch the two fields together. As a practitioner, I have witnessed wrap-around insurance and catastrophe bonds, which offer features of tradable securities in highly illiquid risk transfer markets and which have been on offer in the London capital markets since the early 1990s. These instruments are usually under cover of private placements and therefore, of use only to a few but large investors.

A monoid is simply a category with a single object and single morphism to itself. This is a surprisingly unifying and powerful gadget in category theory proper.
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58 This is not a definition of a functor, which we have sketched above in another footnote. It is unfortunate but because of the limitation of space, the intriguing intricacies of covariant and contravariant functors will not be examined here. I merely indicate without further comment that covariant and contravariant functors would make for excellent conceptual models in the design of operational risk programmes.

59 I have contended that there are no such things in the law as forgetful functors, but I have a more relaxed attitude when it comes to having forgetful functors in the law and finance universe. See Tanega (2012).

60 For the “Orderly Liquidation Authority”, see Dodd-Frank Act (2010).

61 For the Securities Exchange Commission’s Whistleblower Incentives and Protection provisions, see 15 USC Section 78u-6 (n.d.). For the Commodities Futures Trading Commission, see 7 USC Section 26 Commodities Whistleblower Incentives and Protection (n.d.).

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